MV-algebra extended

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Extended abstract

In this paper, we propose extensions of affine classical linear algebra(ACL-algebra) and many-valued algebra(MV-algebra), the emphasis is being on the latter. MV-algebras are algebraic models of Łukasiewicz $\aleph_0$-valued logic(Ł$_{\aleph_0}$) and that was established long back in 1958 by C.C. Chang. MV-algebra is recently studied from the angle of fuzzy set theory. Also it is investigated in connection with substructural logics. CL-algebras are algebraic models of multiplicative additive fragment of linear logic(MALL). Linear logic was introduced by J. -Y. Girard and in A.S. Troelstra's terms 'is interesting from a logical point of view, and potentially of considerable interest for computer science'. Affine CL-algebra is a special case of CL-algebra. Logic corresponding to this algebra is MALL with one structural rule weakening. Interconnection between Łukasiewicz logic, linear logic and other substructural logics has been an important topic of research in recent years [1].

The issue of extension or embedding of an algebraic structure may be considered to be significant purely from algebraic stand point. But in this paper, along with the algebraic studies we have delved into the implication of such extensions in the logical studies as well. Also, connections of the new logical systems with the existing ones, viz., MALL, affine MALL and Ł$_{\aleph_0}$ have been investigated. In [1], a large class of algebras corresponding to various substructural logics are studied. Among these algebras there are many generalizations or extensions of MV-algebras initiated from different motivations. The algebras proposed in this paper are different from those in [1]. Logics corresponding to these extensions are also substructural. Our proposed extended logic (EMVL1) may be placed between the logic MALL and Ł$_{\aleph_0}$. Also, extended affine MALL may be placed in between MALL and affine MALL.

The system Ł$_{\aleph_0}$ and the corresponding algebra(MV-algebra) have found widespread applications in fuzzy set theory. We shall indicate in this paper that there are some inherent conceptual difficulties in connecting

MV-algebras with the philosophy of fuzzy sets. These difficulties are however overcome if extended MV-algebras are taken in place of MV-algebras.

Significance of the unit 1 of fusion operator, i.e., monoidal operator in logic is not very clear. Here in our proposed algebras, we are trying to understand the meaning of 1. In fact, 1 may be thought of as the first accepted truth and after that the truth increases and ultimately attends the absolute truth $\top$ which is the lattice top. In a CL-algebra $0 \leq 1$ or $1 \leq 0$ or these elements may be incomparable. But in the intended interpretation of the fuzzy logic that we are presenting here, 0 plays the role of first false. This is also a reason for taking $0 \leq 1$.

We define an extended affine CL-algebra (EACL-algebra) as a CL-algebra $\mathcal{X} \equiv < X, \wedge, \vee, \perp, \multimap, *, 1, 0 >$ in which $0 \leq 1$ and $< \{x \in X : 0 \leq x \leq 1\}, \wedge, \vee, 0, \multimap, *, 1, 0 >$ is an affine CL-algebra.

Also two types of extensions of MV-algebras(EMV-algebra) viz., EMVA1 and EMVA2 are defined.

An extended many-valued algebra 1 (EMVA1) $\mathcal{X} \equiv < X, \wedge, \vee, \perp, \multimap, *, 1, 0 >$ is a CL-algebra where $0 \leq 1$ and $< M, *, \sim, 0, 1 >$ is an MV-algebra, where $M = \{x \in X : 0 \leq x \leq 1\}$.

An extended many-valued algebra 2 (EMVA2) is an EMVA1 with the additional property that the underlying set $X = L \cup M \cup U$, where $L = \{x : \perp \leq x \leq 0\}$ and $U = \{x : 1 \leq x \leq \top\}$ and L, U are linear.

Logics(sequent calculi) corresponding to the extended affine CL-algebra and one of the extensions of MV-algebras EMVA1 are introduced. Soundness and completeness of these logics with respect to the corresponding algebras are also established.

Like fuzzy set theories on MV-algebras, it is now possible to develop a similar theory on EMV-algebras. Fuzzy set theory based on MV-algebra, i.e., bold fuzzy set theory satisfies law of excluded middle and law of contradiction. These properties do not match with the expectations to the notion of vagueness. Rather the desirable properties are obtained in this fuzzy set theory. In this theory, we have law of approximate contradiction ($\widetilde{A} \otimes \widetilde{A}^c \subseteq \widetilde{\theta}$) and law of approximate excluded middle ($\widetilde{A} \oplus \widetilde{A}^c \supseteq \widetilde{I}$). This follows from the fact that in a CL-algebra $x* \sim x \leq 0$ and so $x+ \sim x \geq 1$ hold. These two properties along with other properties of bold fuzzy set theories are at par with our expectation.

References

1. N. Galatos, P. Jipsen, T. Kowalski and H. Ono : *Residuated Lattices: An algebraic glimpse at Substructural Logics*; Elsevier (2007).