Lecture 1: Tutorial on relation algebras (1.5 hours)

Relation algebras are abstract algebras defined by axioms laid down by Tarski in 1941. The axioms were intended to characterise the *algebras of binary relations* on a set, endowed with the boolean operations such as union and complement, a constant denoting the identity (equality) relation, and functions identifying the converse of a binary relation and the composition of two binary relations.

Relation algebras and algebras of binary relations have some direct applications, for example in computer science. They are also a part of *algebraic logic*, connected to cylindric algebras and algebraisations of first-order logic. However, the subject matter is so fundamental that relation algebras are a breeding ground for general theorems and proof methods that themselves have applications in other areas, such as modal and temporal logic. They are also of interest in their own right.

In this introductory tutorial, we will examine algebras of binary relations, define relation algebras, and compare the two. A *representation* of a relation algebra A is an isomorphism from A onto an algebra of binary relations. The representable relation algebras are of great importance, and we will outline some key results about them.

We will look at *duality* for relation algebras, involving atoms, complete representations, atom structures, complex algebras, canonical extensions, and MacNeille completions. Some examples of relation algebras will be given.