Game Dynamics in Extensive Form

Dietmar Berwanger

Laboratoire Spécification et Vérification
CNRS & ENS Cachan, France

Abstract

One appealing justification of Nash equilibrium is by interpreting equilibria as limits of a dynamic strategy improvement process [4]. The basic notion of evolutionarily stable strategies, formalised by Maynard Smith and Price in [6], has proved to be a fundamental tool for explaining the evolution of social conventions as coordination mechanisms [1] and for analysing the robustness of equilibrium refinement concepts [5, 7].

In the framework of evolutionary game theory, strategies are typically assumed to be completely *mixed*, that is, each agent has a continuum of strategic alternatives. For logical games, and in many scenarios involving computational systems, this is not an appropriate assumption.

In this talk, we propose an approach to capture the process of choosing among *pure* strategies in games with a *discrete* strategy space by modelling the evolutionary dynamics as games in extensive form. Our model is based on infinite path-forming games over graphs.

Formally, we represent the dynamics of a game for n players (in normal form) by a directed graph where nodes correspond to pure strategy profiles and arcs to transitions between two profiles triggered by a strategy switch of one or more players. On this graph, the players play an meta-game by interacting to form a possibly infinite path with long-term utilities given by the discounted cumulative payoff earned from the original game. Thus, we regard the dynamic decision-making process in the (repeated) normal-form game as a variant of a mean-payoff game [2, 8].

Our graph-game model generalises the basic mean-payoff game model of Ehrenfeucht and Mycielskski [2] into two directions: it involves n players and there are no procedural rules: in each state, any player may change his strategy. Hence, we deal with games of infinite duration that are not zero-sum and involve a certain kind of imperfect information. As the theory of such games is still under development, our exposition will remain speculative in large parts.

We illustrate basic properties of the model, showing that for certain classes of games, it promotes Nash equilibria. On the other hand, we argue that, stable outcomes correspond, in general, to *orbits* of strategy switches rather than steady states. Moreover, it turns out, that such orbits may involve switches that are locally non-profitable to reach

an outcome that is more efficient than any equilibrium in pure strategies. Thus, our analysis departs from the more traditional methods based on the best-response graph.

The primary objective of our investigation is to shed light on mechanisms of coordination and endogenous coalition formation in discrete games. A promising related approach is the recent study of Goemans, Mirrokni, and Vetta [3] which proposes a cogent measure for the lack of coordination in a game by analysing game dynamics via random walks on the best-response graph.

By founding our analysis on the theory of automata and infinite games on graphs, we expect to benefit of well-established methods for manipulating (sets of) strategies of a regular structure. These methods may be helpful, in particular to investigate the connection between iterated elimination of dominated strategies with forward and backwards induction.

Finally, we can view the transformation of a game —interpreted by its normal form— into the meta-game of its dynamics —corresponding to an extensive form— as a logical operation which provides access to the internal structure of a game. The fixed-points of this operation represent thus strategically stable outcomes with an extensive structure.

References

[1] K. Binmore, *Game Theory and the Social Contract*, MIT Press, Volume 1: *Playing Fair* (1994), Volume 2: *Just Playing* (1998).

[2] A. Ehrenfeucht and J. Mycielski, *Positional strategies for mean payoff games*, International Journal of Game Theory, 8 (1979), pp. 109–113.

[3] M. X. Goemans, V. S. Mirrokni, and A. Vetta, *Sink equilibria and convergence*, in Proc. 46th Annual IEEE Symposium on Foundations of Computer Science (FOCS 2005), IEEE Computer Society, 2005, pp. 142–154.

[4] G. J. Mailath, *Do people play Nash equilibrium? Lessons from Evolutionary Game Theory*, Journal of Economic Literature, 36 (1998), pp. 1347–1374.

[5] R. Selten, *Evolutionary stability in extensive two-person games*, Mathematical Social Sciences, 5 (1983), pp. 269–363.

[6] M. J. Smith and G. R. Price, *The logic of animal conflict*, Nature, 246 (1973), pp. 15–18.

[7] E. van Damme, *Stable equilibria and forward induction*, Journal of Economic Theory, 48 (1989), pp. 476–496.

[8] U. Zwick and M. Paterson, *The complexity of mean payoff games on graphs*, Theoretical Compututer Science, 158 (1996), pp. 343–359.