Strategic reasoning in compositional games

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1 Abstract

The central innovation introduced by game theory is its strategic dimension. A player's environment is not neutral, and she expects that other players will try to out-guess her plans. Reasoning about such expectations and strategizing one's own response accordingly constitutes the main logical challenge of game theory.

Evaluation games have been long used by logicians to define the semantics of various logics. For instance, in first order logic, given a structure $\mathfrak{A}$ and a formula α, *verifier* claims that $\mathfrak{A} \models \alpha$ whereas *falsifier* claims that this is not true. The game positions are subformulas of α. Every disjunct is associated with the verifier who picks one of the disjuncts whereas a conjunct is associated with the falsifier. Similarly, existential and universal quantifiers are associated verifier and falsifier respectively. For negation the players switch roles. It can be shown that in this two player zero sum game, the verifier has a winning strategy iff $\mathfrak{A} \models \alpha$.

To logically reason about games, one would like to present games in a compositional manner. Inspired by evaluation games, the natural game operators would be choice, sequential composition, dual and iteration of sequential composition. Game logic [Par85] is a logic to reason about such determined two person zero sum games. Semantics of the logic is defined in terms of neighbourhood functions which represents the outcomes players can enforce in the game. The logic makes assertions about composing neighbourhoods or abilities of players. The role of strategies in game logic is limited to just expressing the abilities of players and strategies themselves do not figure in the logical framework.

Strategic reasoning

Under the assumption that players are perfectly rational, reasoning about existence of strategies suffice since such players will always be able to employ their best possible choice of action. However in many practical situations, players are computationally bounded agents. Such agents employ bounded memory strategies which depend on their expectations about the behaviour of other players. Typically players start the game with some prior assumptions and revise their assumptions based on the history information of the play.

In this work, joint with R. Ramanujam, we look at how to incorporate the notion of strategic response of a players to other players' moves in the compositional framework. This can be thought of as a first step in developing a logical

mechanism for reasoning about strategizing of players based on their expectations. We consider the case where atomic games are presented as normal form games.

The logic

Syntax:

- $\Gamma := (g, \eta) \mid \xi_1; \xi_2 \mid \xi_1 \cup \xi_2 \mid \xi^*$
- $\Phi := p \in P \mid \neg\alpha \mid \alpha_1 \vee \alpha_2 \mid \langle\xi\rangle^{\forall}\alpha.$

where g is a normal form game and η is a set of plays in g.

Neighbourhood semantics: In abstract games, the neighbourhood relation is part of the model. However, when games have structure, the neighbourhood relation should be built in a such a way that the choices of players restricts the game under consideration. We take models of the logic to be Kripke structures from which this relation is synthesised.

It can be shown that the logic admits a complete axiomatization and that the satisfiability problem is decidable. As mentioned earlier, we view this study as initial: an appropriate mechanism to capture the notion of expectation and to revise these expectations in the compositional framework needs to be formulated.

On a related note, one can also look at strategic reasoning with respect to a fixed game structure. This is in contrast with the game structure being dictated by the formulas of the logic. Alternating temporal logic [AHK02] has evolved as an influential logic to reason about such games. Here the game structure is presented as a graph where each node is associated with a *single* normal form game. ATL can therefore be thought of as a temporal logics for games rather than a dynamic logic.

References

[AHK02] R. Alur, T. A. Henzinger, and O. Kupferman. Alternating-time temporal logic. *Journal of the ACM*, 49:672–713, 2002.

[Par85] R. Parikh. The logic of games and its applications. *Annals of Discrete Mathematics*, 24:111–140, 1985.