Game Dynamics in Extensive Form

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Evolution in repeated games

0,1	2,1
1,2	0,3
1,1	2,0

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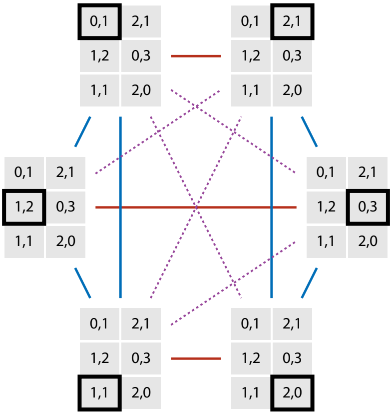
Justification of Nash Equilibrium:

- definite state
- selective pressure

▶ sequential dynamics

Project: Reveal extensive structure from the normal-form representation of a game.

Game dynamics graph



Game Γ in normal form:

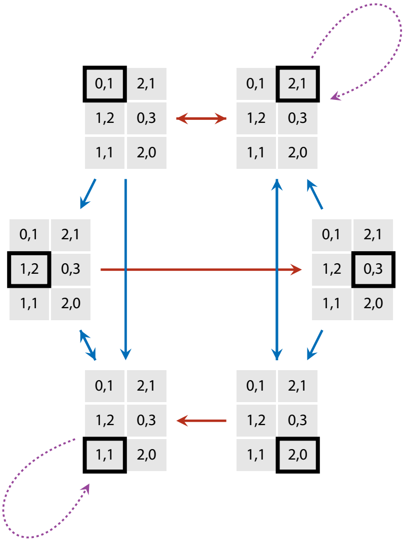
- n players
- strategy sets S^i -- finite
- utility $u^i : S \to \mathbb{N}$

▶ Game dynamics graph $G(\Gamma)$:

- nodes: profiles in S
- edges: switches $s \to s'$ by any subset of players

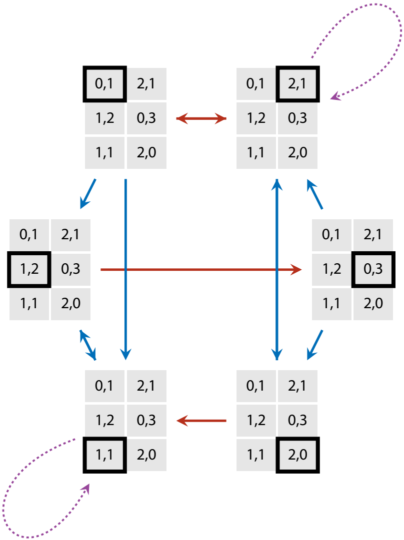
*We consider **pure** strategies.*

Best-response dynamics



Greedy walks
converge to Nash Equilibrium
if well-founded
e.g., in potential games.

Best-response dynamics



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What if not?

Sink equilibria [Vetta & al, 2005]

[Goemans, Mirokni, Vetta 2005]

Sink: terminal connected component of best-response graph.

- Price of sinking -- social cost of lack of coordination vs price of anarchy.

$$\frac{\Sigma u^i(\text{opt})}{\Sigma u^i(\text{worst SinkEq})} \geqslant \frac{\Sigma u^i(\text{opt})}{\Sigma u^i(\text{worst NashEq})}$$

Theorem. The price of anarchy can underestimate the price of sinking by a factor of n.

Consequences for convergence speed of random best-response walks.

Salience of the definite state

Example: Tragedy of the commons - variant

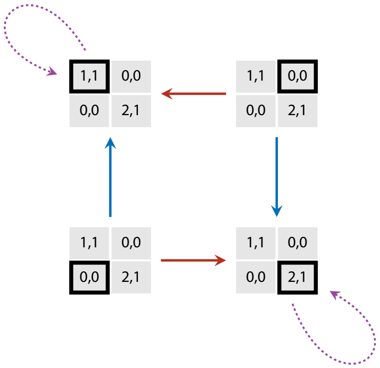
- each player has one responsible and n irresponsible strategies
  - responsible strategies guarantee 1 Rp
  - irresponsible strategies pay off 2 Rs for one player
  - all other irresponsibles -- 0 Rs
- who wins depends on all chosen strategies.

  Pareto optimum: n+1 • Nash Eq: $n+\epsilon$ • Sink Eq: 2

- Effect hides when mixing strategies

  relies on **perfect information** about the current state

Strategising equilibrium selection



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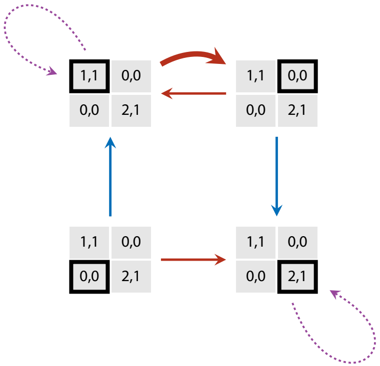

Efficient orbit supported by a non-greedy switch

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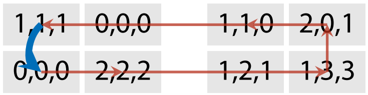
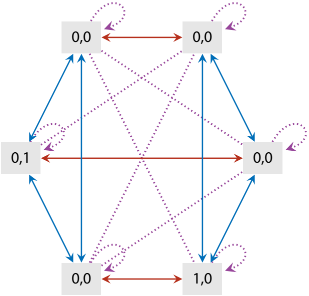


Dynamics metagame



- Game graph: $G(\Gamma)$;
- utility: cumulative
  - mean payoff, discounted
- perfect information
  - no procedural rules
  - fair tie breaking

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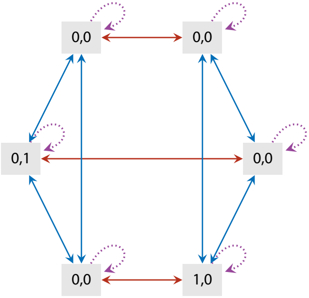

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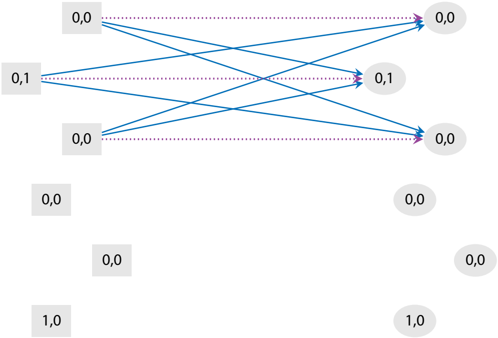

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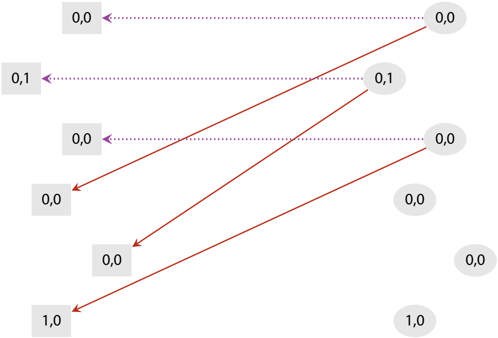

Theorem. [Ehrenfeucht, Mycielski]
Mean-payoff zero-sum games are determined with memoryless strategies.

► feasible outcomes ◄

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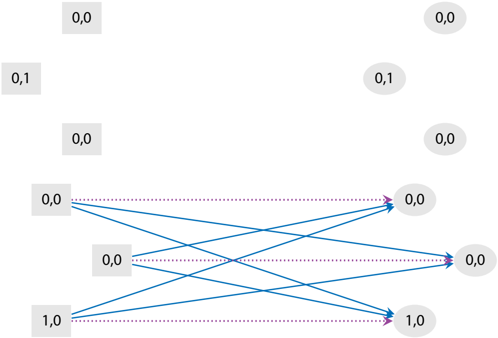

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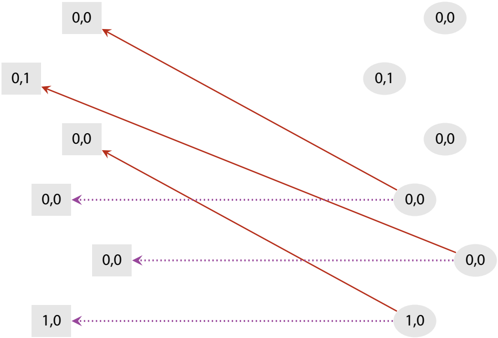

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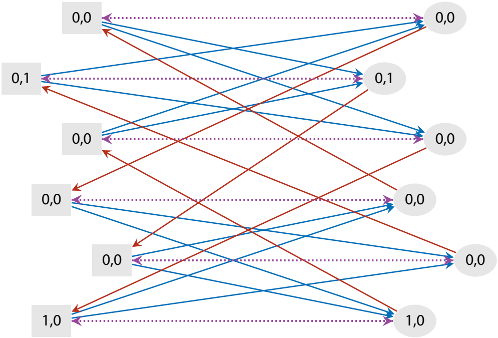

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Challenges

- Games of **infinite duration**:
  - non-zero sum
  - more than two players
  - order of moves
- Folk Theorem

Applications

- Extensive games of perfect information:
  best response to path signalling
  - backwards-inductive outcome
  - subgame-perfect equilibria
- Symmetric games:
  regret minimisation, full signalling
  - iterated elimination of weakly dominated strategies

Conclusion

- Lifting strategic games to extensive meta-games
  - infinte games with numeric payoffs (can be perturbed)
  - plausible when playing with automata
  - captures some standard concepts
- Outlook:
  - Special structure of metagames
  - Value iteration
  - Incomplete information, dynamic objects