Towards a formalization of Lewis' context-dependent notion of knowledge in Dynamic Epistemic Logic

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David Lewis (1941 – 2001)

Work on philosophy of language, philosophy of mind, metaphysics, epistemology, and philosophical logic

Elusive Knowledge in Australasian Journal of Philosophy, 1996, Vol. 74, pp. 549-567

Classic definition of knowledge

Given an agent S and a proposition p, we say:

S knows that p if and only if S has eliminated all possibilities where not-p.

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2. Fallibilism: If you allow that knowledge that p can be achieved despite eliminating all possibilities where not-p the term knowledge is derived from all its content. What does "knowledge despite uneliminated possibilities of error" mean?

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Given an agent S and a proposition p, we say:

S knows that p if and only if S has eliminated *all* possibilities where not-p –Psst!– Except for those possibilities that we are properly ignoring.

Lewis' rules of inclusion

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1. Rule of Actuality
2. Rule of Belief
3. Rule of Resemblance

Lewis' rules of exclusion

1. Rule of Reliability
2. Rule of Method I
3. Rule of Method II
4. Rule of Conservatism
5. Rule of Attention

Lewis' claim

Given a knowledge claim ϕ the rules determine which are the relevant possibilities and which are the irrelevant possibilities, the ones we can properly ignore.

The Rule of Attention: "it is more a triviality than a rule"

When we say that a possibility is properly ignored, we mean exactly that; we do not mean it could have been ignored. Accordingly, a possibility not ignored at all is ipso facto not properly ignored. What is and what is not being ignored is a feature of the particular conversational context.

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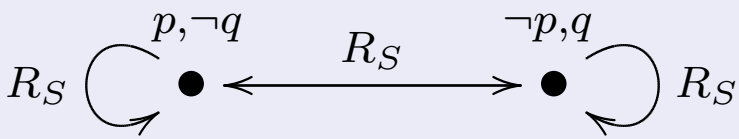

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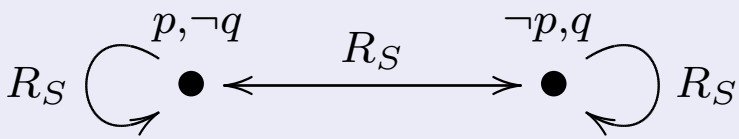

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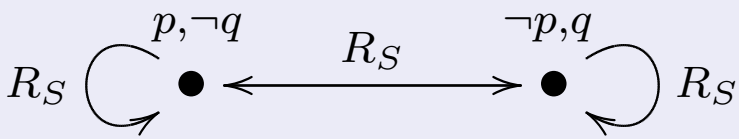

An agent Q enters: "Isn't John in Madrid?"

The Rule of Attention decrees that this possibility has to be considered. S cannot properly ignore it.

- Unattended possibilities coming into play: How do we model it?
- One solution is finite dialogue modelling.
- We 'know' in advance what will happen. This fixes a domain we should consider.

Notation

- Set of agents: $\mathcal{I} = \{i_0, \ldots, i_n\}$
- Set of propositional variables: $P = \{p_0, \ldots, p_k\}$
- Set of actions: $A = \{a_0, \ldots, a_l\}$

Definition (Dialogue)

Stage	Sentence	$p \in P$	Actions	Change of Relevance	Change Access. Relation
0					
1	$\sim\sim\sim$	$X^1 \subseteq P$	$\langle a_0, \ldots, a_k, r_0, \ldots, r_l \rangle$	$\langle \{R_i^1 \mid i \in I\} \rangle$	$\langle \{S_i^1 \mid i \in I\} \rangle$
2	$\sim\sim\sim$	$X^2 \subseteq P$	$\langle a_0, \ldots, a_k, r_0, \ldots, r_l \rangle$	$\langle \{R_i^2 \mid i \in I\} \rangle$	$\langle \{S_i^2 \mid i \in I\} \rangle$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
n	$\sim\sim\sim$	$X^n \subseteq P$	$\langle a_0, \ldots, a_k, r_0, \ldots, r_l \rangle$	$\langle \{R_i^n \mid i \in I\} \rangle$	$\langle \{S_i^n \mid i \in I\} \rangle$

- We assume that the Relevance set of an agent i is accumulative, i.e., $X_i^m \subseteq X_i^n$, if $m < n$.
- Some actions imply a change of relevance. For instance, an announcement ϕ.
- After ϕ is announced at line k, and variables $q_0, \ldots, q_s$ occur in ϕ, we require that $r_{I,q_0}, \ldots, r_{I,q_n}$ are in k
- In general, $Rel : A \to POW(R)$
- If an action a is in line k, then all of $Rel(a)$ is in line k.
- If $i \in I$ and $r_{i,p}$ in line n, then $p \in R_i^n$

Model

- A domain $W = 2^n$, where n is the number of propositions in P;
- Accessibility relation $S_i^m \subseteq W \times W$, for every $i \in I$.

Definition

Given a relevance set $R_i^n \subseteq P$, we define an equivalence relation $\sim_{n,i}$ on W:

$$w \sim_{n,i} w' \Leftrightarrow \forall p \in R_i^n (p \in w \leftrightarrow p \in w')$$

How to evaluate knowledge of i at n?

Definition

We define the accessibility relation $\hat{S}_i^n$ on $W/\sim_{n,i}$ as follows:

$$[w]\hat{S}_i^n[v] \iff wS_i^nv,$$

where we let $w_{n,i}$ be w with all values of $p \notin R_i^n$ set to false.

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Definition

Let

$$\mathbb{M} := \langle W, \{S_i^n \mid i \in I\}, \{R_i^n \mid i \in I\}\rangle$$

$$\mathcal{M}_i^n := \langle W/\sim_{n,i}, \{\hat{S}_i^n \mid i \in I\}\rangle$$

Definition (Semantics)

$$\mathbb{M}^n, w \models p \qquad \Leftrightarrow p \in w$$

$$\begin{aligned} \mathbb{M}, w \models K_i p \qquad & \Leftrightarrow \\ \mathcal{M}_i^n, w \models K_i p \quad & \Leftrightarrow \forall v : [w]\hat{S}_i^n[v] \Rightarrow \mathcal{M}_i^n, v \models p \end{aligned}$$

Example

- $I = \{Bill, Marie, James\}$
- $p :=$ John is in Paris, $l :=$ John is in London, $m :=$ John is in Madrid, $a :=$ John is in Amsterdam

Stage	Sentence	$R^n \subseteq P$	Actions	Change of Relevance	Change Access. Relation
0					
1	Bill: p, l, or m?	$\{p, l, m\}$	$\langle\rangle$	$r_1 = \{p, l, m\}$	$S^1_{b,m,j}$
2	Marie: not (L or P)!	$\{p, l, m\}$	$[!(\neg(p \,\&\, l)]$	$r^2 = r^1 = r^2$	$S^2_{b,m,j}$
3	Marie: Isn't he in a?	$\{p, l, m, a\}$	$\langle\rangle$	$r^3 = a$	$S^3_{b,m,j}$
4	James: not Madrid	$\{p, l, m, a\}$	$[!\neg m]$	$\langle\rangle$	$S^4_{b,m,j}$

Conclusions

- Lewis' rule of Attention has a dynamic character.
- This is more complicated then Lewis might have anticipated.