MEANING, CHOICE and ALGEBRAIC SEMANTICS of SIMILARITY BASED ROUGH SET THEORY

A. Mani

Member, Calcutta Mathematical Society
a.mani@member.ams.org
Homepage: http://amani.topcities.com

International Conference on Logic and Applications, 2009

ABSTRACT

Both algebraic and computational approaches for dealing with similarity spaces are well known in generalized rough set theory. However, these studies may be said to have been confined to particular perspectives of distinguishability in the context. In this research, the essence of an algebraic semantics that can deal with all possible concepts of distinguishability over similarity spaces is progressed. Key to this is the addition of choice-related operations to the semantics that have connections to modal logics as well. In this presentation, I will focus on a semantics based on *local clear distinguishability* over similarity spaces.

Outline

INTRODUCTION

- **Approximation Space**: $S = \langle \underline{S}, R \rangle$, where $\underline{S}$ is a set and R is an equivalence.
- If $A \subset S$, $A^l = \bigcup\{[x]\, ;\, [x] \subseteq A\}$ and $A^u = \bigcup\{[x]\, ;\, [x] \cap A \neq \emptyset\}$ are the lower and upper approximation of A respectively
- **Pawlak's Knowledge Semantics**: If $\underline{S}$ is a set of attributes, then sets of the form A^l and A^u represent clear and definite concepts. If Q is another stronger equivalence on $\underline{S}$, then the state of the knowledge encoded by $\langle \underline{S}, Q \rangle$ is a *refinement* of that of $S = \langle \underline{S}, R \rangle$.
- **Tolerance Approximation Space** (TAS): $S = \langle \underline{S}, T \rangle$, where T is a tolerance relation.
- **Granules**: All approximations are built up from these indivisible (relative the specific rough semantic domain) units. In case of TAS, the most used concept of granulation is the set of T-relateds: $[x] = \{y : (x, y) \in T\}$.

MORE TAS

- **Blocks**: A block $B \subseteq S$, is a subset that is maximal w.r.t $B^2 \subseteq T$. Tolerances are fully determined by their blocks.
- Cattaneo'98: $A^{l*} = \{x\,;\, (\exists y)\,(x, y) \in T,\, [y]_T \subseteq A\}$; $A^{u*} = \{x\,;\, (\forall y)\,((x, y) \in T \longrightarrow [y]_T \cap A \neq \emptyset)\}$. This approach has a reasonable algebraic semantics (BZ-algebra and variants) associated.

Proposition

For any subset A, $A^l \subseteq A^{l*} \subseteq A \subseteq A^{u*} \subseteq A^u$ ☐

- The BZ-algebra and variants do not capture all the possible ways of arriving at concepts of distinguishability over similarity spaces. Other related approaches have more shortcomings.
- In subjective terms, reducts are minimal sets of attributes that preserve the quality of classification. An important problem is in getting good scalable algorithms for the computation of the different types of reducts (or supersets that are close to them).

LOCAL CLEAR DISTINGUISHABILITY

- Blocks are the natural generalization of equivalence classes to tolerances, where as sets of T-related elements or the sets of conservatively T-related elements do not qualify.
- A subset $A \subseteq S$ has the role of a context initiator in the natural extension of knowledge semantics from AS. Approximations of A can correspond to clear concepts only when they are constituted of nonintersecting granules.
- By the **local clear distinguishability** principle (LCP), we mean the requirement that definite objects generated by context initiators should be made up of nonintersecting granules.
- $A = \{a_1, a_2, \ldots a_n\}$: LCP + FIFO can determine an unique lower approximation (a union of a maximal set of disjoint blocks in A). Choice is determined by the choice of the order.
- Minimal set of disjoint blocks containing A may not exist. So we can define the upper approximation in many ways.

FORMAL DEFINITIONS

- For $A \subseteq P$ - a POSET, let $L(A) = \{x ; (\forall a \in A)\, x \leq a\}$ and $U(A) = \{x ; (\forall a \in A)\, a \leq x\}$. $\lambda : \wp(P) \mapsto P$ will be said to be *lattice-coherent* with $<$ if $a \leq b$ then $\lambda(L(a, b)) = a$ and $\lambda(U(a, b)) = b$
- A *choice function* χ on a set S, is a function $\chi : \wp(S) \longmapsto S$ s.t. $(\forall x \in S)\, \chi(\{x\}) = x$ and $(\forall A \in \wp(S))\, \chi(A) \in A$
- $\mathbb{S}$ - collection of all blocks of T. If $E, B \in \wp(\mathbb{S})$ then $E \prec B$ iff $E \subseteq B$ and E is a subcollection of disjoint blocks.
- Lower Relativisation Form the collection $\mathbb{S}(A)$ of all blocks included in A
  Lower Clarification-1,2 $\mathbb{LS}(A)$ - subcollections of mutually disjoint elements in $\mathbb{S}(A)$. Order these by inclusion and let the set of maximal elements be $\mathbb{LS}_M(A)$.

DEFINITIONS (Continued)

Choice We will assume that we have a choice function $\lambda : \wp(\wp(\mathbb{S})) \mapsto \wp(\mathbb{S})$ that is lattice-coherent with the $\prec$ order on the collection $\wp(\mathbb{S})$.

Lower Choice $\bigcup \lambda(\mathbb{LS}_M(A))$ - *0-lower approximation* (A^{l0}) of A.

Primitive Lower Choice $\lambda(\mathbb{LS}_M(A))$ - *primitive lower approximation* of A

Lateral Lower Choice $\bigcup \mathbb{S}(A)$ - *lateral lower approximation* $(A^{\check{l}})$ of A.

Upper Relativisation $\mathbb{S}_u(A)$ - Set of blocks that intersect A.

Upper Clarification-1,2 $\mathbb{US}_m(A))$ - Set of minimal elements in the set of subcollections of mutually disjoint blocks in $\mathbb{S}_u(A)$ each of whose unions contains A.

Upper Choice $\bigcup \lambda(\mathbb{US}_m(A))$ will be called the *0-upper approximation* of A. It will be abbreviated to A^{u0}. If $\mathbb{S}_u(A)$ is empty, then take A^{u0} to be undefined.

Primitive Upper Choice $\lambda(\mathbb{US}_m(A))$ - *primitive upper approximation* of A

Lateral Upper Choice $\bigcup \mathbb{S}_u(A)$ - *lateral upper approximation* $(A^{\check{u}})$ of A

THEOREM

All of the above approximations are well-defined and satisfy the following properties:

(a) $(A^{l0})^{l0} = A^{l0} \subseteq A^{\check{l}}$; $A^{l0} \subseteq (A^{l0})^{u0}$

(b) $(A^{u0})^{l0} \overset{w}{=} A^{u0} \overset{w}{=} (A^{u0})^{u0} \subseteq A^{\breve{u}}$; For terms p, q, $p \overset{w}{=} q$ iff $(\forall x \in dom(p) \cap dom(q))p(x) = q(x)$ (of course w.r.t an interpretation)

(c) $(A \subseteq B \longrightarrow A^{l0} \subseteq B^{l0})$

(d) $(A \subseteq B, A \subseteq A^{u0}\, B \subseteq B^{u0} \longrightarrow A^{u0} \subseteq B^{u0})$

(e) $(A \subseteq B \longrightarrow A^{\check{l}} \subseteq B^{\check{l}}, A^{\breve{u}} \subseteq B^{\breve{u}})$

(f) If $A^{l0} = A = A^{\check{l}}$, then A is necessarily a union of disjoint blocks.

(g) If A^{u0} exists, then $A^{l0} \subseteq A^{l} \subseteq A^{l*} \subseteq A^{l\theta} \subseteq A \subseteq A^{u\theta} \subseteq A^{u0} \subseteq A^{u*}$ else, $A^{l0} \subseteq A^{l} \subseteq A^{l*} \subseteq A^{l\theta} \subseteq A \subseteq A^{u\theta} \subseteq A^{u*}$

(h) If A is a block, then $A^{l0} = A = A^{\check{l}}$, or $A^{u0} \neq A^{l0}$ and $A^{\breve{u}} \neq A^{u0}$

More Properties

Theorem

If we define the operations $\sim$, $\ominus$ *over the power set* $\wp(S)$ *via (the latter being a partial operation that is defined only when* A^{u0} *is)*
$\sim A = S \setminus A^{\breve{u}} \quad \ominus A = S \setminus A^{u0}$, *then* $A \subseteq \sim\sim A$, *but in general* $A \nsubseteq \ominus\ominus A$, *even when the RHS is defined.*
For any $A, B \subseteq S$, *let* $A \curlyvee B = (A^{u0} \cup B^{u0})^{u0}$ *(if defined) and* $A \curlywedge B = (A^{u0} \cap B^{u0})^{l0}$ *(if defined) then the following hold for the partial operations:*

(a) $A \curlyvee A \overset{w}{=} A^{u0u0}; (A \curlyvee B)^{l0} = A \curlyvee B$

(b) *If* $A = \{a, b\}$, $a \neq b$ *is not in any block, then* $A^{l0} = \emptyset$, $A^{\breve{l}}$ *and* A^{u0} *are undefined, while* $A^{\breve{u}}$ *is a union of at least two blocks.*

(c) $A \curlywedge A \overset{w}{=} A^{u0}; \; A \curlywedge B \overset{w}{=} B \curlywedge A$

(d) $(A \subseteq B \subseteq B^{u0} \longrightarrow A \curlyvee B = B^{u0})$

Definition: Pre-Essential λ-Rough Partial Algebra

$\Xi(S) = \left\langle \underline{\wp(S)|\sigma}, \leq, \curlyvee, \curlywedge, \sqcup, \varovee, \varowedge, L_0, U_0, \check{L}, \breve{U}, \sim, \ominus, [\emptyset], [S] \right\rangle$ that has been constructed as follows from a TAS S:

- For any set $A \in \wp(S)$, if A^{u0} is defined let $\upsilon(A) = (A^{l0}, A^{u0}, A^{\check{l}}, A^{\breve{u}})$, else let $\upsilon(A) = (A^{l0}, A^{\check{l}}, A^{\breve{u}})$
- Let $(A, B) \in \sigma$ if and only if $\upsilon(A) = \upsilon(B)$
- Then form the quotient $\wp(S)|\sigma$
- Define $L_0([A]) = [A^{l0}]$, $U_0([A]) = [A^{u0}]$ if defined
- On the quotient, let $[A] \leq [B]$ iff $A^{l0} \subseteq B^{l0}$ and $A^{u0} \subseteq B^{u0}$ (if defined) and $A^{\check{l}} \subseteq B^{\check{l}}$ and $A^{\breve{u}} \subseteq B^{\breve{u}}$. Strict version of inequality: $\lneq$
- Define $[A] \varovee [B] \stackrel{\text{def}}{=} [A^{u0} \cup B^{u0}]$ if defined
- Define $[A] \varowedge [B] \stackrel{\text{def}}{=} [A^{u0} \cap B^{u0}]$ if defined
- Define $[A] \curlyvee [B] \stackrel{\text{def}}{=} U_0([A] \varovee [B])$ if defined
- Define $[A] \curlywedge [B] \stackrel{\text{def}}{=} L_0([A] \varowedge [B])$ if defined

Definition (Continued)

- Define $\breve{U}([A]) \overset{\text{def}}{=} [A^{\breve{u}}]$, $\breve{L}([A]) = [A^{\breve{l}}]$
- Define $[A] \sqcup [B] \overset{\text{def}}{=} [A \cup B]$
- Define $[A] \sqcap [B] \overset{\text{def}}{=} [A \cap B]$
- Define $\sim [A] \overset{\text{def}}{=} [S \setminus A^{\breve{u}}]$
- Define $\ominus[A] \overset{\text{def}}{=} [S \setminus A^{u0}]$ if defined

More Operations on $\Xi(S)$:

- For any x, if x is the class of a single block, then let $s(x) = x$, else $s(x) = \emptyset$.
- For any x, if x is the class of a 2-element subset that is not in any block, then let $t(x) = x$, else $t(x) = \emptyset$.
- $IU(x)$ if and only if $U_0(x) = U_0(x)$. Note that U_0 is a partial operation. Further we will write $IU(a, b, ..)$ for $IU(a), IU(b), \ldots$
- $IN(x)$ if and only if $\ominus x = \ominus x$.

The algebra formed by adjoining the additional operations and predicates $(\lneq, s, t, IU, IN)$ to $\Xi(S)$ will be termed an *Essential λ-Rough Partial Algebra* and denoted by $\maltese(S)$.

ABSTRACTION: AER

Abstract Essential λ*-Rough Partial Algebraic System* (AER): $S = \left\langle \underline{S}, \leq, \sqcup, \owedge, L_0, U_0, \breve{L}, \breve{U}, \sim, \ominus, t, 0, 1, (2, 2, 2, 1, 1, 1, 1, 1, 1, 1, 0, 0)\right\rangle$ s.t. [(we assume that the operation $\sqcup$ is complete and the derived operations $\ovee$, $\curlyvee$, $\curlywedge$, s and derived predicates $\lneq$, IU, IN are defined via:

- If the RHS is defined then and only then $x \ovee y = U_0(x) \sqcup U_0(y)$
- $x \leq y, \neg(x = y)$ if and only if $x \lneq y$
- $x \curlyvee y \overset{w}{=} U_0(x \ovee y)$
- $x \curlywedge y \overset{w}{=} L_0(x \owedge y)$
- $\forall y(y \lneq x \longrightarrow L_0(y) \lneq L_0(x))$ and $L_0(x) = U_0(x) = x$ if and only if $s(x) = x$, else $s(x) = 0$
- $IU(x)$ iff $U_0(x) = U_0(x)$. Further we will write $IU(a, b, ..)$ for $IU(a), IU(b), \ldots$
- $IN(x)$ if and only if $\ominus x = \ominus x$.]

(*) $x \curlyvee y \overset{w}{=} y \curlyvee x$; $x \curlywedge y \overset{w}{=} y \curlywedge x$; $x \curlyvee x \overset{w}{=} U_0U_0(x)$

(*) $x \curlywedge x \overset{w}{=} U_0(x)$; $L_0L_0(x) = L_0(x)$; $(IU(x) \longrightarrow U_0(x) \leq U_0U_0(x))$

AER (Continued)

- $L_0(x) \leq \check{L}(x) \leq x\,\check{U}(x)$; $(IU(x) \longrightarrow x \leq U_0(x))$; $L_0\check{L}(x) \leq \check{L}(x)$
- $\check{L}L_0(x) = L_0(x)$; $\check{L}\check{L}(x) = \check{L}(x)$; $(IU(x) \longrightarrow L_0(x) \leq U_0L_0(x))$
- $L_0(x) \leq U_0L_0(x)$; $\check{U}(x) \leq \check{U}\check{U}(x)$
- $(IU(x) \longrightarrow x \leq U_0(x) \leq \check{U}(x) \leq \check{U}U_0(x) \leq \check{U}\check{U}(x))$
- $(x \leq y, IU(x, y) \longrightarrow x \curlywedge y = U_0(x) = x \owedge y, x \curlyvee y = U_0(y) = x \ovee y)$;$(IU(x, y, a, b), x \leq y, a, \leq b \longrightarrow x \curlywedge a \leq y \curlywedge b)$
- $(IU(x, y, a, b, x \ovee a, y \ovee b), x \leq y, a, \leq b \longrightarrow x \curlyvee a \leq y \curlyvee b)$
- $(x \leq y \longrightarrow L_0(x) \leq L_0(y), \check{U}(x) \leq \check{U}(y), \check{L}(x) \leq \check{L}(y))$
- $(x \leq y, IU(x) \longrightarrow U_0(x) \leq U_0(y))$
- $t(x) = x$ iff $\neg(IU(x)), s(\check{U}(x)) = 0,\ L_0(x) \lneq x,$
  $(\forall y)(y \lneq x \longrightarrow ty = 0)$ and
  $(0 \lneq a, b, c \lneq x \longrightarrow a = b \text{ or } b = c \text{ or } c = a)$
- $(IU(x) \longrightarrow \sim \ominus x \leq \ominus \sim x)$; $\sim x \leq \sim L_0(x)$; $\sim 0 = 1$; $\sim 1 = 0$
- $(IU(\sim\sim x) \longrightarrow x \leq \sim\sim x)$; $(IU(x) \longrightarrow \sim U_0(x) \leq \sim x)$

AER (Cont'd!)

- $\sim \breve{U}(x) \leq \sim x$; $(IU(x) \longrightarrow \sim \breve{U}(x) \leq \sim U_0(x)$
- $\sim x \leq \sim \breve{L}(x) \leq \sim L_0(x)$; $(IN(x) \longrightarrow \ominus x \leq \ominus L_0(x))$
- $(IU(\sim x), IU(x) \longrightarrow \sim \ominus x \leq \ominus \sim x)$; $\ominus 0 = 1$; $\neg IN(1)$
- $(IU(x) \longrightarrow \ominus x \leq \ominus L_0(x))$; $(IN(x) \longrightarrow \ominus \breve{U}(x) \leq \ominus x, \ominus U_0(x) = \ominus x)$
- $(\forall x)\, 0 \leq x \leq 1$
- $\forall y((0 \leq y \leq x \longrightarrow y = 0 \text{ or } y = x), \longrightarrow \bigvee(s(z) = z, x \leq z))$, where $\bigvee$ indicates disjunction over the entire set S
- $(s(x) = x, x \lneq y \longrightarrow s(y) = 0)$
- $(s(x) = x, y \lneq x \longrightarrow s(y) = 0)$
- $x \sqcup y = y \sqcup x$; $x \sqcup x = x$;
- $(x \leq y \longrightarrow x, y \leq x \sqcup y)$; $\breve{(U)}(x) \sqcup \breve{U}(y) \leq \breve{U}(x \sqcup y)$

REPRESENTATION THEOREM

Theorem

Given an abstract essential λ-rough partial algebra S there exists a TAS and a choice perspective that ensures that its algebraic semantics is isomorphic to S.

Proposition (Implication-Like Operations)

In $\maltese(S)$, if we define $x \rightsquigarrow y = (\sim x) \sqcup (\breve{U}y)$ and $x \rightarrowtail y = (\ominus x) \sqcup (U_0 y)$ if defined, then the following hold:

(a) $(IU(x) \longrightarrow x \rightarrowtail x = 1); (IU(x) \longrightarrow U_0(x) \leq 1 \rightarrowtail x)$

(b) $(IU(x, y) \longrightarrow U_0(x) \leq x \rightarrowtail (y \rightarrowtail x)); (IU(x) \longrightarrow x \rightarrowtail 0 = \ominus(x))$

(c) $x \rightsquigarrow x = 1; x \leq (1, \rightsquigarrow x); x \rightsquigarrow 0 = \sim x$

(d) $(x \rightsquigarrow y) \sqcup (x \rightsquigarrow z) \leq (x \rightsquigarrow (y \sqcup z))$

(e) $(x \rightsquigarrow z) \sqcap (y \rightsquigarrow z) \leq (x \sqcup y) \rightsquigarrow z$ ☐

SUMMARY

- We have developed a similarity-based rough set theory using blocks as granules. Choice functions have been integrated into the semantics in an intuitive way.
- For the algebraic semantics, we have a representation theorem.
- Some other types of rough set theory over TAS can be reduced to special cases. We hope to demonstrate the reduction of the BZ-variants and Bitten semantics. But the reduction problem is not as important as relating the choice functions to modal semantics for example.
- Reducts can be computed with greater ease in the theory.
- The developed semantics can be related/combined with modal Tarski algebras.

FURTHER DIRECTIONS

- Develop sequent calculi for the semantics
- Connections with Modal Tarski Algebras
- Classify the semantics for an improved theory of knowledge
- In bitten rough set theory (Slezak, D. and Wasilewska, P.), the granules used are the sets of T-relateds, but the upper approximation is obtained by removing the negative region of the complement of the set. Two different algebraic semantics have been developed by the present author in [6], one of which is applicable in abstract granular rough set theory as well. Connections of the latter with the present approach can be of some interest.

References

- Cattaneo, G., Ciucci, D.: Algebras for Rough Sets and Fuzzy Logics, in: *Transactions on Rough Sets II* (A. Skowron, J. F. Peters, Eds.), vol. 2, Springer Verlag, 2004, 208–252.
- Celani, S.: Modal Tarski Algebras, *Reports on Math. Logic*, **39**, 2005, 113–126.
- Chajda, I., Niederle, J., Zelinka, B.: On Existence Conditions for Compatible Tolerances, *Czech. Math. J*, **26**, 1976, 304–311.
- Mani, A.: Choice-Based Generalized Rough Semantics and Modal Logics, *Submitted*, 2008, 45 pp.
- Mani, A.: Esoteric Rough Set Theory-Algebraic Semantics of a Generalized VPRS and VPRFS, in: *Transactions on Rough Sets* (A. Skowron, J. F. Peters, Eds.), vol. VIII, Springer, 2008, 182–231.
- Mani, A.: Algebraic Semantics of Similarity-Based Bitten Rough Set Theory, *Submitted*, 2008, 16pp.
- Slezak, D., Wasilewska, P.: Granular Sets - Foundations and Case Study of Tolerance Spaces, in: *RSFDGrC 2007, LNAI 4482*, vol. 4482, Springer, 2007, 435–442.
- Snasel, V.: Lambda Lattices, *Math. Bohemica*, **122**(3), 1997, 267–272.

ESSENTIAL λ-ROUGH PARTIAL ALGEBRA: $\maltese(S)$

Theorem

All of the fundamental and derived operations of a pre-essential λ-rough partial algebra are well defined and all of the following hold:

(a) *If x is a class corresponding to a union of disjoint blocks then $L_0x = \check{L}x = x$ and conversely.*

(b) *If $x = U_0x$, then x is a class corresponding to a union of disjoint blocks, but the converse need not hold in general.*

(c) *If x is a class generated by a single block, then $L_0x = U_0x = x = \check{L}x$*

(d) *If for a class x, $(\forall y)(y \lneq x \longrightarrow y \neq L_0(y) \neq U_0(y))$ and $L_0(x) = U_0(x) = x$, then x is the class corresponding to a single block and conversely*

(e) *If for a class x, $(\forall y)(y \lneq x \longrightarrow L_0(y) \lneq L_0(x))$ and $L_0(x) = U_0(x) = x$, then x is the class corresponding to a single block and conversely*

(f) *If for a class x that does not correspond to that of a single block, ($L_0(x) = [\emptyset]$ or $L_0(x)$ corresponds to a single block) and $U_0(x)$ is undefined, then x is a class that corresponds to a set that contains a two element set that is not in any block of T.*

(g) $(U_0(x) = U_0(x) \longrightarrow \ominus(x) \leq \ominus(L_0(x)))$

Optional: PROOF of REPRESENTATION THEOREM

Proof.

Our abridged proof has three components (roughly). The first concerns the reconstruction of the tolerance approximation space, the second part of the choice perspective and the third part of compatibility builds into the first two. Let S be an AER as in the above definition. Then the statements 14, 22 – 27 and the definitions of $\lnsim$, s, t, ensure that we can reconstruct a tolerance T on a set K (corresponds to 1) by the representation theorem of tolerance relations (see [3]). $\sqcup$ is needed for getting the set K in a easier way. We do not have a full representation here.

Both the operations L_0 and U_0 permit the isolation of the choice function used as

- Blocks can be identified through the function s.
- Blocks can be combined via $\sqcup$
- Maximal unions of mutually disjoint blocks can be identified
- Any union of blocks is constructible $\square$

$\square$

Optional:MODAL CONNECTIONS

Modal Tarski Algebra [2]: $A = \langle \underline{A}, \Rightarrow, \Box, 1\rangle$ s.t

- $1 \Rightarrow a = a$; $a \Rightarrow a = 1$
- $(a \Rightarrow (b \Rightarrow c) = (a \Rightarrow b) \Rightarrow (a \Rightarrow c)$
- $(a \Rightarrow b) \Rightarrow a = (b \Rightarrow a) \Rightarrow a$
- $\Box 1 = 1$; $\Box(a \Rightarrow b) \leq \Box a \Rightarrow \Box b$; $a \leq b$ iff $a \Rightarrow b = 1$

Construction: S - TAS

- $\Delta(S)$ - elements are unions of 'a complement of a block and a subset of the block'
- Define $U \Rightarrow V = (\underline{S} \setminus U) \cup V$ $X = \langle \Delta(S), \Rightarrow, \underline{S}\rangle$ *is a Tarski Subalgebra of* $\wp(S)$

Proposition: If the rough equivalence σ of the previous section is applied on $\Delta(S)$, then the resulting classes are of two types: those that correspond to complements of a block and those that correspond to unions of complement of a block and a little more. $\Box$

MORE REFERENCES

- [BK] Banerjee, M., Khan, A.: Multiple-Source Approximation Systems, in: *Proc RSKT 2008 Chengdu, China* (Grzymala-Busse, J. et al, Eds.), Springer 2008,80–87.
- [BNNSW] Bazan, J., Nguyen, S., Nguyen, H., Synak, P., Wroblewski, J.: Rough Set Algorithms in Classification Problem, in: *Rough Set Methods and Applications* (L. P. et al., Ed.), Physica Verlag, Heidelberg, 2000, 49–88.
- [BZ] Breslov, R., Stavrova, A., Zapatrin, R.: Topological Representation of Posets, *ArXiv. Math*, **GN/0001148**, January 2000, 1–11.
- [CG98] Cattaneo, G.: Abstract Approximation Spaces for Rough Set Theory, in: *Rough Sets in Knowledge Discovery 2* (L. Polkowski, A. Skowron, Eds.), Physica Heidelberg, 1998, 59–98.
- [IM] Inuiguchi, M.: Generalisation of Rough Sets and Rule Extraction, in: *Transactions on Rough Sets-1* (J. F. Peters, A. Skowron, Eds.), vol. LNCS-3100, Springer Verlag, 2004, 96–116.
- [KM] Komorowski, J., Pawlak, Z., Polkowski, L., Skowron, A.: Rough Sets – a Tutorial, in: *Rough Fuzzy Hybridization* (S. K. Pal, Ed.), Springer Verlag, 1999, 3–98.
- [KC] Kryszkiewicz, M., Cichon, K.: Towards Scalable Algorithms for Discovering Rough Set Reducts, in: *Transactions of Rough Sets-1* (J. F. Peters, A. Skowron, Eds.), vol. LNCS-3100, Springer Verlag, 2004, 120–143.