

The Fabric of Commands

Ordinary discourse : --

- Argumentative
- Non-argumentative

Argumentative discourse may be formal or informal :--

- Informal – Visual images found in art, design, advertising, world wide web

Formal –

- (1) Fictions -factual truth is not the purpose. So usually there is no proof involved, though they may have good internal logic.
- (2) Conditional statements –they are not arguments, though they often attempt to justify something.
- (3) Explanations– some may have deductive character, but the purpose is not to prove.
- (4) Commands –a series of commands may be tested for logical consistency. they are neither true nor false. they are often hidden conditionals, viz. “Work hard to succeed”.

Present paper concentrates on arguments containing commands.

there are many constructions having imperative intents:--

1. Necessity imperatives—
Commands- “Arise, awake and stop not till the goal is reached’
Warning- “ Look before you leap”
2. Possibility imperatives – “Stay late if you like”
3. Expressive imperatives – “Have a good time’
4. Informative imperatives – “Press the f4 key to view the slide show”

Now commands combined by logical connectives like “either. or..”, “if..then..” are initially more puzzling than commands having categorical form. But commands of any form is commonly accepted as expressing a “prescription” as distinct from “proposition”, though the possibility of their crossing over is not ruled out.

Prescriptions can be impersonal or personal, unsatisfiable (let $2+3=6$) or unviolable (let $2+3=5$) or many other types. Presently we concentrate on the logical status of prescriptions as constituents of arguments. They have two primary features:

1. they have performatory use.
2. they occur in a context (both unconditional and conditional prescriptions)

Historical background :--

Aristotle (practical syllogism having conclusion as an action)

Stoics (commands are distinguished from propositions)

Leibnitz (modalities)

Hume (normative-descriptive distinction)

Ernst Mally (If $\vdash (B \rightarrow C)$ then $\vdash OB \rightarrow OC$. Mally was the first one to speak of the Principle of Inheritance of Obligation while developing a formal logic for normatives.

Logicians take different stands in explaining the inferential property of an argument involving commands at par with that property of an inference of classical propositional logic. Many logicians speak of the observability of patterns of "entailments" between imperatives viz.

Jorgensen, Ross, von Wright, Hare, Segerberg to name some of them.

There are two approaches— reductionists and non-reductionists. The reductionist approach tries to translate command sentences into declarative one, thereby bringing them within the sphere of classical logic. The other approach rules out any sort of isomorphism between imperative and classical logic. The strength comes from the inapplicability of the rules of propositional logic to imperative logic. Consider the following argument:--

Browse the net and give me the infos.

So, browse the net.

This can be easily explained in like manner of the conjunction elimination rule of propositional logic. But it is not easy to treat all cases in the same way :--

Hit the target.

(Ross, 1941)

So, hit the target or run away.

It is difficult to account for its validity as it is counter-intuitive in nature. Now, in order to discuss the concept of validity, it is necessary to deal with the command sentences and their logical status. We can speak of three possible values so to speak of command sentences. Consider a conditional prescription :--

“ If you love him, pray for him” is

- (i) satisfied if you love him and pray for him,
- (ii) violated if you love but don't pray for him,
- (iii) avoided if you don't love him, no matter whether you pray for him or not.

“you pray for him” (satisfaction proposition)

“you don't pray for him” violation proposition)

“you don't love him” (avoidance proposition)

So prescription is an ordered pair of satisfaction and violation propositions respectively. it is an ordered pair of logically incompatible propositions , because no prescription can be both satisfied and violated.

Attempts are found to define satisfiability in the following way.

“A command is satisfied if some appropriate propositional description is true at some point in future, but it can only be true if the presuppositions of that propositional description holds,, and it is precisely the presuppositions that capture the satisfiability conditions of the imperative.”

This “built-in” notion of satisfiability fails to be adequate, because imperatives often are obeyed after the time of utterance. So it can not be called “valid” at the time of utterance.

Following Peter B.M.Vranas propositions are hereafter to be treated as sets(sets of all possible worlds) and all the connectives accordingly.

In case of a prescription, the satisfaction, violation and avoidance sets form a partition of the set of all possible worlds, they are mutually exclusive and jointly exhaustive. Thus, given any two, the third one is the complement of the union of the remaining two. so a prescription can be specified with reference to any two sets, without being confused with two-valued logic.

In case of conditional prescription context is its condition. The context is the union of its satisfaction and violation sets. The avoidance therefore is the negation of its context. In case of unconditional prescription, there is no condition, but there is context. There is a tendency to deny the possibility of avoidance in this context, but I like to argue against it later on.

prescription = $\langle S, V \rangle$

context = (SuV)

avoidance = $\sim (SuV)$

Negation—

unconditional prescription— help him

negation— don't help him

you don't help him (satisfied)

you help him (violated)

you remain indifferent (avoided)

conditional prescription- if you love him, help him.

negation– you love him, but not help him.

you love him, but not help him. (satisfied)

you love him and help him. (violated)

you don't love him. (avoided)

Definition—The negation of the prescription with satisfaction set S and violation set V is the prescription with satisfaction set V and violation set S .

prescription $I \rightarrow \langle S, V \rangle$

negation-- $\sim \langle S, V \rangle = \langle V, S \rangle$

Apart from this total negation, there are satisfaction-negation and violation-negation

$\sim_s \langle S, V \rangle = \langle S^c, S \rangle$ (superscript denotes complementation)

$\sim_v \langle S, V \rangle = \langle V, V^c \rangle$

some problems—

- (i) The rule of double negation does not hold here. Starting with conditional prescription, one may end up with unconditional prescription.
- (ii) Different prescriptions can have same \sim_s and same \sim_v
 - If you do A, do B
 - If you do B, do A
 - \sim_s of both-- Don't do both A and B.

Conjunction—

Unconditional prescription : ‘ “Trust me” and “Touch me” ’

s— both are satisfied

v— one or both are violated

a— denial of all acquaintance

Conditional prescription : ‘ “If you love me, trust me” and “if you love me, touch me” ’.

s and v are the same as the previous one, avoided if both are avoided (if you don't love me)

In case of conjunctive conditional prescriptions, having two different contexts —

context— $(C \cup C')$; $v— (V \cup V')$

$s— (S \cup S') \ \& \ \sim(V \cup V')$

In case of conjunctive conditional prescriptions(p and p') having two contexts (taking s', v' and a' for satisfaction, violation and avoidance of second prescription) :

$$[c(p \ \& \ p')] = (c \cup c')$$

$$[v(p \ \& \ p')] = (v \cup v')$$

$$[a(p \ \& \ p')] = (a \cap a')$$

$$[s(p \ \& \ p')] = (s \cup s') \sim (v \cup v')$$

Definition- The conjunction of two prescriptions is the prescription whose context is the union of the contexts of the conjuncts and whose violation set is the union of the violation sets of the conjuncts.

conjunctions of two prescriptions :

$$\langle S, V \rangle \& \langle S', V' \rangle = \langle (C \cup C') \sim (V \cup V'), V \cup V' \rangle$$

$$= \langle (S \cup S') \sim (V \cup V'), V \cup V' \rangle$$

(here $C = S \cup V$ and $C' = S' \cup V'$)

Disjunction—

Unconditional- “Write to me or talk to me”.

s- if at least one disjunct is satisfied

v- if both are violated

a— denial of all acquaintance

Conditional- “If you love me, write to me , or if you don’t love me, write to me”

Vranas suggests that it is the same as the prescription expressed by “write to me” (whether or not you love me)

Hence it loses the status of a conjunction.

In case of two disjuncts having two distinct contexts, the definition is as follows :

Definition- The disjunction of two prescriptions is the prescription whose context is the union of the contexts of the disjuncts and whose satisfaction set is the union of the satisfaction sets of the disjuncts.

disjunction of two prescriptions :

$$\begin{aligned} \langle S, V \rangle \vee \langle S', V' \rangle &= \langle S \cup S', (C \cup C') \sim (S \cup S') \rangle \\ &= \langle S \cup S', (V \cup V') \sim (S \cup S') \rangle \end{aligned}$$

Conditionals—

The consequent of a conditional is a prescription:

If he trusts you, help him.

S—he trusts you and you help him.

V—he trusts you but you don't help him.

A— he does not trust you.

Definition-The conditional whose antecedent is the proposition P and whose consequent is the prescription with satisfaction set S and violation set V is the prescription whose satisfaction set is the intersection of P with S and whose violation set is the intersection of P with V .

$$P \rightarrow \langle S, V \rangle = \langle P \cap S, P \cap V \rangle$$

Biconditional—

Biconditional consisting of the conjunction of two conditionals each having a prescription P and a prescription I:

Help him if and only if he trusts you.

s- P true and I satisfied ,or P false and I violated.

v- P true and I violated, or P false and I satisfied.

Definition— The biconditional $P \leftrightarrow I$ of the proposition P and the prescription I is the conjunction of the conditionals $P \rightarrow I$ and $P^c \rightarrow \sim I$.

$$\begin{aligned} P \leftrightarrow \langle S, V \rangle &= (P \rightarrow \langle S, V \rangle) \& (P^c \rightarrow \langle V, S \rangle) \\ &= \langle P \cap S, P \cap V \rangle \& \langle P^c \cap V, P^c \cap S \rangle \\ &= \langle (P \cap S) \cup (P^c \cap V), (P \cap V) \cup (P^c \cap S) \rangle \end{aligned}$$

Now let us concentrate on the question of validity of arguments involving commands. WE may start with some presupposed model of obedience and bindingness to account for validity. Keeping in mind that satisfaction is one of the alternative values of a prescription, this paper tries to accommodate satisfaction as the criterion of validity. Hence it is obligatory to show the applicability of the same. The important point to note is that satisfaction can be framed in the model of bindingness , with some essential modifications. But so far as obedience is concerned, it has to be argued seriously in favour of satisfaction as against obedience. Once the charges against satisfaction can be handled adequately, it would be easier to present satisfaction as a better candidate.

To proceed in this venture, let me introduce obedience , in a general sense, in a nutshell.

The case of an unconditional prescription “Inform her” is trivial with the giving or not giving the information, no question of avoidance being there.

conditional prescription-If you find her, inform her.

S-you find her and inform her.

V-you find her but don't inform her.

A- you don't find her.

This obedience-condition is not fulfilled only if the prescription is violated. So obedience is to be understood as non-violation:

Obedience=Satisfaction or Avoidance.

Applying this rule to pure imperative argument, we get the result--

If you meet me, inform me and help me.

So, if you meet me, inform me.

It is obedience-valid in this way:-

[(meet me, inform me, help me) \vee (don't meet me) \rightarrow
(meet me, inform me) \vee (don't meet me)]

Argument against this position may be presented from two perspectives:

- (1) formalization of obedience
- (2) interpretation of satisfaction-validity

Formalization of Obedience

Formalization of obedience, presented so far, having avoidance as its constituent poses great problem both for unconditional and conditional imperatives. Unconditional prescription is ordinarily interpreted as that which does not have avoidance set and so obedience-validity is more or less equated with satisfaction-validity in this context. I intend to say that an unconditional prescription can equally accommodate the possibility of avoidance. And hence the so-called formalization of obedience can be reviewed in both the contexts. Let us remember the interpretation of obedience :

A prescription = (**s v a v v**)

obedience = **~v**

= **s v a**

Let us take an example of unconditional prescription :

“Shut the door”

Here the definition of obedience in terms of the distinction can be evaluated with reference to two disjuncts(**s v a**) separately :

- (1) The first disjunct : There is serious problem whether apparent satisfaction or so-called “carrying out” the command does really mean obedience. Let us suppose that the utterance of this command is followed by the event of shutting of the door, yet it could be false that it is obeyed. We can offer different possible analyses of the event as follows :
 - (a) The addressee might not have listened to the command at all, but was going to close the door anyway and did it of her own accord.**
 - (b) The hearer might have heard the command, and intending to ignore it walking by the doorway accidentally shut it.**
 - (c) The hearer might have wanted to do as she was told, but when she actually shut it, she might have done it because of the dust which was coming through the door.**

Now, while (a) and (c) may be borderline cases , but (b) is a clear case where mere “carrying out” of the action does not constitute obeying.

(2) The second disjunct :Serious objection comes from a case of avoidance.Let us take another example :

A person is given the command **“Speak the Truth”** and the person keeps silent.

Here the command is avoided because the person did not allow any occasion of speech. It is a clear case of voluntary avoidance which has sufficient empirical reason to be considered as a case of disobedience. Hence the definition fails.

In case of conditional prescription, avoidance seems to have much safe-guard in being a constituent of the said definition :

If you love him, pray for him = $L \rightarrow P$

obedience = $(L \ \& \ P) \vee \sim L$

Here the first disjunct of this non-exclusive disjunction is by itself strict non-violation. Again it is debatable whether the second disjunct leads to non-violation/obedience in the same way as the first one. Now avoidance indicates the falsity of the antecedent of a conditional prescription and thereby leads to obedience of the command. So it never directly causes obedience. The resulting obedience is apparent or vacuously fulfilled. This apparent naïve presence of avoidance yields a jerking effect when it plays the same role in leading to disobedience :

$$\begin{aligned} \text{Avoidance} &= (\sim L \rightarrow P) && \{ \text{obedience} \} \\ &= (\sim L \rightarrow \sim P) && \{ \text{disobedience} \} \end{aligned}$$

In view of what has been said , obedience may be comfortably understood as neither violated nor avoided, which is the same as satisfied.

A prescription = (**s v v v a**)

obedience = ($\sim v$ & $\sim a$)

= **S** (by applying separately the two conjuncts).

So the case can be settled in favor of satisfaction-validity instead of obedience-validity.

Formalization of Satisfaction-validity

The argument from I(premise) to I'(conclusion) is satisfaction-valid exactly if s(satisfaction of premise) entails s'(satisfaction of conclusion).

Firstly it is debatable whether the concept of entailment (as found in classical deductive logic) is applicable in case of imperative argument.

Secondly, the distinction between strong and weak sense of validity is very pertinent in case of imperative argument, but it is vague to speak of strong and weak sense of entailment .

Now the main weakness of satisfaction as a criterion of validity under this formulation may be shown with the help of two counter-examples:

(1) If it rains, close the window.

So, if it rains and thunders, close the window.

(2) If you wake up and see a burglar, call the police.

So, if you wake up, call the police.

Let us consider one after the other. But before that, we may start with the preliminary uses of satisfaction principle.

Satisfaction-principle may be defined in different logics in the following way :

- A. Sentential Logic—A formula is satisfiable in terms of an interpretation under which the formula is true.
- B. Predicate Logic---A formula is satisfiable if and only if it is true under at least one interpretation. Eventually a formula is valid if and only if it is satisfied under every interpretation viz. $p \vee \sim p$.
- C. Imperative Logic--- A formula is satisfiable if and only if a reason supports a prescription. A formula is valid means it is in force.

Among the two previous examples the first one is said to be intuitively valid, but satisfaction-invalid while the second is intuitively invalid but satisfaction-valid. The drawback of satisfaction-validity, according to this opinion, depends heavily on intuitions thereby running the risk of conflicting intuitive results of different people. On the other hand, even if intuitions are concerned, obedience-validity is well equipped to handle these cases, and it is shown with the help of two premises:

- (1) If a pure imperative argument is an instance of strengthening the antecedent, then it is strongly valid.**
- (2) If it is strongly valid, then it is intuitively valid.**

The distinction between strong and weak sense of validity can be drawn in this way :

A pure imperative argument is strongly valid exactly if, necessarily, every reason that strongly supports the conjunction of the premises of the argument also strongly supports the conclusion of the argument.

A pure imperative argument is weakly valid exactly if, necessarily, every reason that weakly supports the conjunction of the premises of the argument also weakly supports the conclusion of the argument.

Taking **R** as “**It rains**”, **C** as “**Close the window**”, **T** as “**It thunders**”, we can have symbolic form of the previous argument(1):

$$\mathbf{R \rightarrow W}$$

$$\mathbf{So, (R \& T) \rightarrow W}$$

The intuitive validity of this argument having strengthened antecedent has been shown in this way :

R → W is interpreted as **[(R & T) → W] & [(R & ~T) → W]**

$$\mathbf{c-(R \& T) \vee (R \& \sim T)}$$

$$\mathbf{\vee-(R \& T \& \sim W) \vee (R \& \sim T \& \sim W) \text{ i.e. } (R \& \sim W)}$$

Now this reduction of violation proposition depends on the closeness of the two contexts of rain and thunder. We may take another example on two unrelated contexts :

If it rains, close the window.



So, if it rains and your work is done, close the window.

Nobody will argue in favour of the intuitive validity of this argument. Nobody will say that the premise is the conjunction of “If it rains and your work is done, close the window” and “If it rains and your work is not done, close the window”. The crux of the issue is, we should bear in mind the distinction between intuitive validity on the one hand, and formal validity of classical propositional logic on the other. This example may easily be cited as a case of formally valid argument, but it is a weak candidate, if at all, as an intuitively valid argument.

The second example (2) is a clear case of intuitively invalid argument. The question is, whether it is satisfaction-valid or not. In order to settle it, satisfaction is to be properly interpreted. For this interpretation, I refer to a general model of bindingness that says “if the premises of an imperative arguments are binding, the so also is the conclusion.

The first thing to note is that the terms “obedience”, “bindingness” etc. advocate the importance of reason in favour of the premises and the conclusion. These ‘reasons’ are facts, which may be moral, legal etc. Undoubtedly imperative arguments are supported moral reasons. Here the relation between reason and prescription is not entailment. So the term “bindingness” appears to be less attractive than “satisfaction”, the former carrying with it the concept of entailment. Let us proceed to make the formalization of satisfaction with the help of other associate definitions.

Definition I : A pure imperative argument is valid i.e. satisfied, exactly if, necessarily, every reason that supports the conjunction of the premises of the argument also supports the conclusion of the argument.

A practical question arises in respect of personal (“you turn off the light”) and impersonal prescription (“let there be light”), because the latter primarily does not approve any possibility of supporting reason. A probable solution can be offered here with the help of an example :

Let the volcano erupt.

The reason – Volcanic eruption can prevent earthquake deaths(remembering that earthquake can be averted if there is volcanic eruption).

Definition II- A reason supports a prescription exactly if it favours the satisfaction over the violation proposition of the prescription.

Now this favouring reason is an empirical phenomenon that can be comparatively stronger than another in the same area. So we have to distinguish between strong and weak sense of supporting reason.

Definition III- A reason strongly supports a prescription exactly if, (1) it favors every proposition which entails the satisfaction proposition of the prescription over every different proposition which entails the violation proposition of the prescription, and (2) it does not favor any proposition which entails the satisfaction proposition of the prescription over any other such possible proposition.

This use of “entailment” does not however contradict earlier rejection of the same, because this relation is here conceived to hold between propositions.

Definition IV– A reason weakly supports a prescription I exactly if, it strongly supports some prescription I* whose satisfaction proposition entails the satisfaction proposition of I and whose context is the same as the context of I.

Now we can return to the previous example (2) of calling police after waking up and seeing a burglar.

$s - (W \& B \& P) \quad ; \quad s' - (W \& P)$

Here the reason that supports s is supposed to be in

favor of the conjunction. So the interconnection of W,B and P are relevant in identifying the supporting reason. Now what is relevant for the collective presence of W,B and P may not be ,if at all, relevant for the presence of w and P. So it is not by mere simplifying the satisfaction-proposition of the premise that we gat the satisfaction-proposition of the conclusion. Hence it is simply unjust to say that the reason that supports (W&B&P) is the same that supports (W&P). Thus it is not satisfaction-valid in any sense. This concludes my argument in favor of satisfaction-validity in the context of imperative argument.

The validity of pure imperative arguments , in respect of the strong and weak senses, can now be tested :

Arguments both strongly and weakly valid

If you go to the market, let me listen to music at home.

So, if you go to the market and meet a friend, let me listen to music at home.

Arguments neither weakly nor strongly valid

(i) Help him.

So help him or beat him.

(ii) If the river overflows, leave the place.

So, if you don't leave the place, let the river not overflow.

Arguments weakly but not strongly valid

Read the book.

If you read the book, comment on it.

So comment on it.

This is the initial introduction of validity principle of conditional imperative. As the disjunctive imperatives are, in a sense, hidden conditionals, so they are automatically covered by this discussion. The task of elaborating the rules of validity for pure imperative arguments however remains for further study in this area.

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