ICM satellite event on logic and set theory

Abstract of talks

Joan Bagaria
Structural Reflection and the Hierarchy of $C(n)$ cardinals

We introduce first a new kind of reflection principle for the universe of all sets: Structural Reflection (SR), which asserts, roughly, that every definable proper class of structures of the same type reflects at some level of the set-theoretic universe. Then we present some results showing the equivalence of various natural forms of SR with the existence of some well-known large cardinals, such as supercompact or extendible. In general, SR turns out to be equivalent to the existence of $C(n)$-extendible cardinals. As a corollary we give new characterizations of Vopenka's Principle in terms of $C(n)$-extendible cardinals. Finally we present some recent applications of SR and $C(n)$ cardinals to Category Theory.

Andrew Brooke-Taylor
Zero-one laws for Fraisse limits over infinite languages

One nice feature of Fraisse limits for finite languages is that there is an associated zero-one law for the finite structures in the corresponding Fraisse class. We show that for Fraisse classes over infinite relational languages that locally are finitary in a suitable sense, there are also corresponding zero-one laws under an appropriate choice of probability measure. Examples of such Fraisse classes are the classes of simplicial complexes and of hypergraphs. We contrast our zero-one law with the known zero-one law of Blass and Harary.

Rob Goldblatt
Elementary Classes Generating Varieties of Complex Algebras

This talk will survey work on connections between the first-order logic of relational structures and the equational logic of their Boolean algebras of subsets. There are many elementary classes of structures whose subset algebras form a variety, i.e. an equationally definable class, of Boolean algebras with additive operators. Moreover, these varieties are "canonical", i.e. closed under certain completions arising from the Jonsson-Tarski generalization of the Stone representation. Examples include representable relation algebras, closure algebras, cylindric algebras, and numerous varieties of modal algebras.

We examine this context by viewing structures as forming a category, under a notion of "bounded" morphism, that is dual to a category of Boolean algebras with operators. The talk will discuss the following question, amongst others.

Is every canonical variety generated by the subset algebras of some elementary class of structures? If Str(V) is the class of structures whose subset algebras belong to variety V, when is Str(V) elementary? If K is an elementary class, under what conditions is K equal to Str(V) for some variety V?

Menachem Magidor

Square like prinicples and Forcing axioms

Cédric Milliet

Groups with few types

A structure is small if it has countably many pure n-types for each integer n. Such structures arise when one whishes to count the number of pairwise non-isomorphic countable models of a given theory. Weakly small structures have been introduced by Belegradek to give a common generalisation of small and minimal structures. A structure is weakly small if for every finite tuple A coming from this structure, there are countably many 1-types over A. We shall show that weakly small algebraic structures behave very much like omega-stable ones, at least locally: in a weakly small group, subgroups which are definable with parameters in a finitely generated algebraic closure satisfy local descending chain conditions. An infinite weakly small group has an infinite abelian subgroup. A nilpotent small group is the central product of a definable divisible group with one of bounded exponent. Every weakly small division ring of positive characteristic is locally finite dimensional over its centre. The Jacobson radical of a weakly small ring is nil, locally nilpotent. Every weakly small division ring is locally modulo its radical the product of finitely many matrix rings over division rings.

Justin Moore

Forcing Axioms and the Continuum Hypothesis

It is known that there is a strongest consistent forcing axiom, namely the forcing axiom for partial orders which preserve stationary subsets of ω_1 (also known as Martin's Maximum or MM). The consistency of this axiom was established by Foreman, Magidor, and Shelah relative to the existence of a supercompact cardinal. Since MM has proved very effective at settling statements which are independent of ZFC, it is natural to ask whether there is an analogous optimal forcing axiom which is relatively consistent with the Continuum Hypothesis.

One precise way to cast this question is whether there are two Π_2 sentences in the language of $(H(\omega_2), \in, \mathrm{NS}_{\omega_1})$ which are each Ω-consistent with CH but which jointly negate CH. This problem is due to Woodin, who showed that the answer is negative if one replaces CH with ZFC. We show that the answer to Woodin's problem is positive and in the process establish that there are two preservation theorems for not adding reals which can not be subsumed into a single iteration theorem. This is joint work with David Aspero and Paul Larson.

Andre Nies

Borel Structures

Continuum size structures occur naturally in analysis, algebra, and other areas. Examples are the additive group of real numbers, and the ring of continuous functions on the unit interval. How about effectiveness constraints on their presentations? A reasonable approach is to require that domain and relations are Borel. Most examples of structures from the areas above have presentations of this kind.

Borel structures were introduced by H. Friedman in 1978. He proved that each countable theory has a Borel model of size the continuum. His main interest, however, was in quantifiers such as "there exists a co-meager set of *x*'s such that ..." Some results were obtained till the late 1990s, for instance results on Borel partial orders by Harrington and Shelah.

The subject was revived by work with Hjorth, Khoussainov, and Montalban (LICS 2008). We were primarily interested in presentations via Büchi automata (which process infinite strings of symbols). For instance, the additive group of reals is Büchi presentable; so is the Boolean algebra B of sets of natural numbers modulo finite differences. All Büchi presentations are Borel. In the LICS paper it was shown that some Büchi presentable structure close to the Boolean algebra B does not have an injective Borel presentation (where each element is represented uniquely). This answered an open question from the theory of automatic structures. It is still unknown whether B itself has an injective presentation.

So far the language was assumed to be countable. Hjorth and I considered the more general case where the language is uncountable but Borel (for instance, the language of a vector space over the reals). In a recent JSL paper we showed that the completeness theorem fails for Borel structures in this wider sense: some complete Borel theory has no Borel model.

I will end the talk with open questions. Woodin asked whether each Borel Scott set is the standard system of a Borel model of PA. Further, does every Borel field Borel embed into a Borel algebraically closed field? If not, this would yield an alternative proof of the result with Hjorth.

Rehana Patel

Classifying Theories of Graphs with a Forbidden Subgraph

Given a graph H, we say that a graph G is H-free if H does not embed into G as a subgraph, induced or otherwise. Cherlin, Shelah and Shi (1999) have shown that for any fixed, finite, connected graph H, the theory of the existentially complete H-free graphs is complete and model complete; further, they give an elegant criterion under which this theory is ω-categorical. The question then arises: given a finite connected graph H, where does the theory of the existentially complete H-free graphs lie within Saharon Shelah's classification spectrum, which is a taxonomy for complete first order theories based on certain syntactic properties? We are especially interested in the region of Shelah's classification consisting of theories that have the so-called n-strong order properties (SOP_n), $n > 2$. Among these, theories with SOP_3 but not SOP_4 are considered the most tractable. I will provide a general condition, related to the Cherlin, Shelah and Shi criterion for ω-categoricity, for the failure of SOP_4, and use this to give an example of an infinite family of graphs H for which the theories of the existentially complete H-free graphs all possess SOP_3 but not SOP_4. I will also discuss partial results and open questions concerning the classification of the theory of existentially complete H-free graphs for an arbitrary finite connected graph H. All definitions will be given.

Kobi Peterzil

O-minimal ingredients in solutions to arithmetic conjectures in Algebraic Geometry

In a seminal paper Pila and Zannier established a beautiful connection between a theorem of Pila and Wilkie on rational points of definable sets in o-minimal structures and arithmetic conjectures about special points in projective algebraic varieties. In their paper they gave a new proof of the Manin-Mumford conjecture (a theorem of Raynaud), and since them Pila was able to prove, using similar ideas, some unknown cases of the analogous Andre Oort conjecture.

In this talk I will outline the Pila-Zannier strategy and focus on the o-minimal ingredients of the proof.

Anand Pillay

Measures in model theory

I will discuss the increasing role of Keisler measures in model theory, and in particular the ubiquity of stable-like measures in certain unstable theories.

Dilip Raghavan

Cofinal types of ultrafilters

A directed set D is said to be **Tukey reducible** to another directed set E, written $D \leq_T E$, if there is a function $f : D \to E$ which maps unbounded subsets of D to unbounded subsets of E. We say D and E are **Tukey equivalent** if $D \leq_T E$ and $E \leq_T D$. The notion of Tukey equivalence tries to capture the idea that two directed posets "look cofinally the same", or have the same "cofinal type". As such, it provides a device for a "rough classification" of directed sets based upon their "cofinal type", as opposed to an exact classification based on their isomorphism type. This notion has recently received a lot of attention in various contexts in set theory. In joint work with Todorcevic, I have investigated the Tukey theory of ultrafilters on the natural numbers, which can naturally be viewed as directed sets under reverse containment. In the case of ultrafilters, Tukey reducibility is coarser than the well studied Rudin-Keisler reducibility (RK reducibility). I will present some recent progress on the Tukey theory of ultrafilters, focusing on the question "under what conditions is Tukey reducibility actually equivalent to RK reducibility?"

Janak Ramakrishnan

Definable linear orders definably embed into lexicographic orders in o-minimal structures

Definable linear orders definably embed into lexicographic orders in o-minimal structures We completely characterize all definable linear orders in sufficiently rich o-minimal structures. Let M be an o-minimal structure expanding a field, for instance the real field. Let $(P, \prec)$ be any definable linear order in M. Then $(P, \prec)$ embeds definably in $(M^{n+1}, <_{\text{lex}})$, where $<_{\text{lex}}$ is the lexicographic order and n is the o-minimal dimension of P. This improves a result of A. Onshuus and C. Steinhorn in the case that M is o-minimal expanding a field.

Denis I. Saveliev

Groupoids of ultrafilters

There exists a natural way to extend the operation of any groupoid (in fact, any universal algebra) to ultrafilters; the extended operation is right topological in the standard compact Hausdorff topology on the set of ultrafilters; the extensions of semigroups are semigroups. Semigroups of ultrafilters are used to obtain various deep results of number theory, algebra, dynamics, etc. The main tool is idempotent ultrafilters. They exist by a general theorem establishing the existence of idempotents in compact Hausdorff right topological semigroups.

Expanding this technique to non-associative groupoids, we isolate a class of formulas such that any satisfying them compact Hausdorff right topological groupoid has an idempotent, and a class of formulas that are stable under passing from a given groupoid to the groupoid of ultrafilters. If a formula belongs to both classes (like associativity), any satisfying it groupoid carries an idempotent ultrafilter. Results on semigroups following from the existence of idempotent ultrafilters (like Hindman's Finite Sums Theorem) remain true for such groupoids.

Another generalization concerns infinitary analogs of these results. The main obstacle here is that non-principal idempotent ultrafilters cannot be σ-additive. We define ultrafilters with two weaker properties (ultrafilters close to $\varkappa$-additive subgroupoids and $\varkappa$-additive ultrafilters close to subgroupoids) and show that their existence suffices to obtain desired infinitary theorems.

Theodore A. Slaman

Structures Recursive in a Random Real

We address the question, "Which structures can be recursively represented relative to a random real?" In joint work with Greenberg and Montalbán, we give both highly non-trivial (non-hyperarithmetic) examples of such structures and also a reasonably sharp upper bound on the complexity of any such structure.

S.M. Srivastava

Stochastic Kripke models

In this talk, we shall use measurable selection theorems to prove several results of independent interest on transition probabilities. As a corollary to our results we show that logical equivalence, bisimilarity and behavioral equivalence are equivalent for stochastic Kripke models over Polish spaces. This extends the classical Hennessy-Milner Theorem in modal logics to stochastic Kripke models. This is a joint work with E. E. Doberkat, Dortmund, Germany.

Wolfgang Thomas

Refining determinacy results for infinite games

In computer science, the theory of infinite games is pursued with an emphasis on algorithmic aspects. This is of considerable interest since infinite games are a natural and useful model of reactive nonterminating computation. We discuss recent results on sharpened versions of determinacy, in which logics or certain types of automata, used for the definition of games, are connected with the presentation of winning strategies in these games. We exhibit several weak logics L interpreted over the domain of natural numbers, for which L-definable games have L-definable winning strategies, whereas for other (mostly more expressive) logics this transfer fails. These results are a motivation to develop a more general understanding of the connection between the descriptive complexities of winning conditions and winning strategies.

Gunnar Wilken

Tracking Chains of Σ_2-Elementarity

As a result of joint work with Tim Carlson we provide a complete arithmetical analysis of the structure $\mathcal{R}_2$, which is a structure of ordinals defined on the basis of Σ_2-elementary substructure, below the least ordinal *a* such that any pure pattern of order 2, as introduced by Carlson (2009), has a covering below a. We show that a is the proof-theoretic ordinal of $KP\ell_0$. As shown by Carlson the collection $\mathcal{P}_2$ of pure patterns of order 2 is well-quasi ordered under coverings. The result presented here shows that this wqo-result is independent of $KP\ell_0$.