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Spatial and Temporal Knowledge Representation

Antony Galton

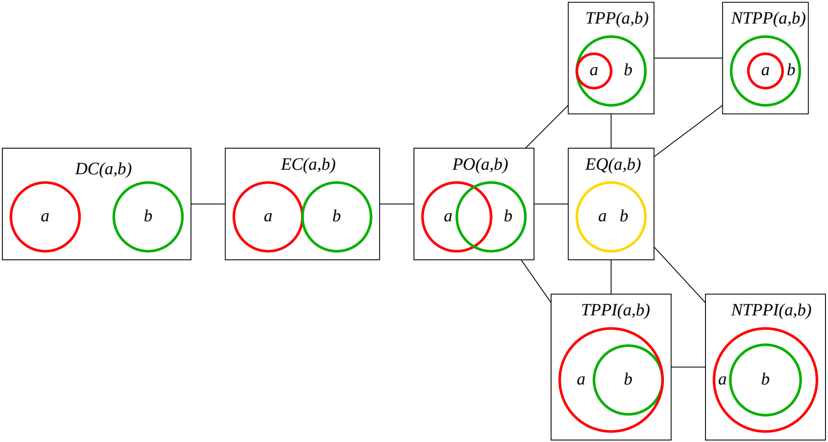
University of Exeter, UK

PART IV: Combining Space and Time

Contents of Part IV

Temporal Interpretation of Conceptual Neighbourhood

Conceptual Neighbourhood Diagram for RCC8

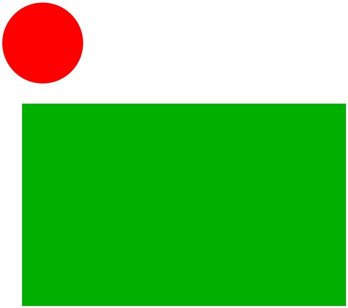


Interpreting the links

The links in the Conceptual Neighbourhood Diagram can be interpreted as representing possible paths of **continuous change**.

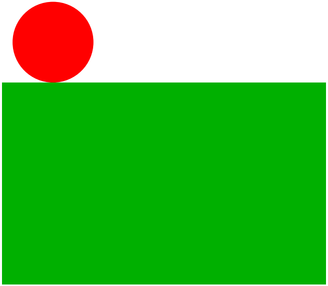
Spatial relations R_1 and R_2 are linked in the diagram if it is possible for two regions which stand in relation R_1 to be transformed, by continuous movement and/or deformation, so that they stand in relation R_2 (or vice versa).

Example: By continuous motion, DC (disconnected) can be directly converted into EC (externally connected).



DC(red,green)

EC(red,green)

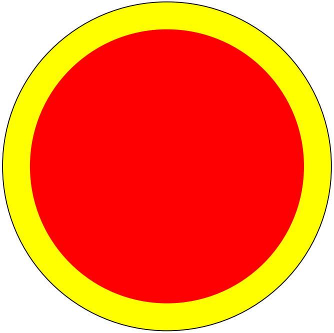


Objects can move and change shape; arguably regions, as portions of space, cannot.

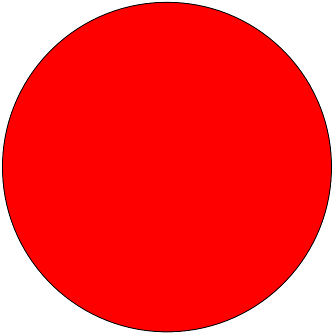
Hence in the previous example, it is natural to regard *red* as an object and *green* as either an object or a region.

Example: By continuous growth, NTPP (non-tangential proper part) can be directly converted into EQ (equal).

(Red is the object, yellow the region.)



NTPP(red,yellow)



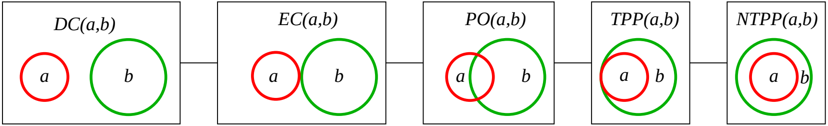
EQ(red,yellow)

A Special Case: Rigid Motion

Special case: Rigid Motion

Assume that the objects/regions are of fixed shape and size. The only spatial changes they can undergo are changes of position (movements). In this case, not all the links in the Conceptual Neighbourhood Diagram represent possible transitions.

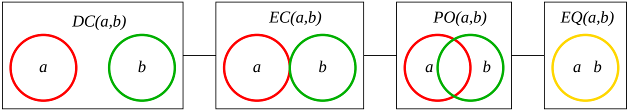
Example: Possible configurations for a circular object (*a*) in the same plane as a larger circular region (*b*). The available RCC8 relations are DC, EC, PO, TPPi, and NTPPi, giving the following Conceptual Neighbourhood Diagram:



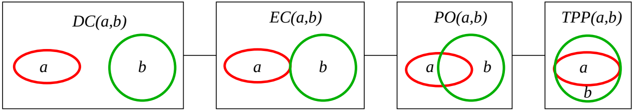
Reference: A. Galton, 'Towards and integrated logic of space, time, and motion', Proceedings of IJCAI'93, pp. 1550–1555.

Rigid Motion — the six possible cases (I)

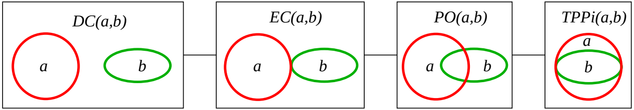
1. *a* and *b* are congruent:



2. *a* can just fit inside *b*:

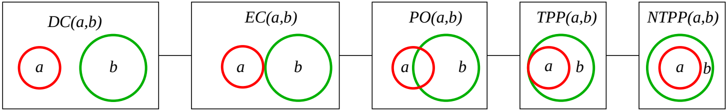


3. *a* can just cover *b*:

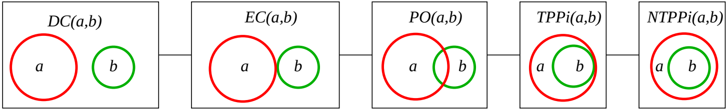


Rigid Motion — the six possible cases (II)

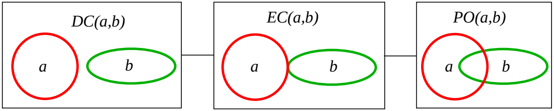
4. *a* can fit right inside *b*:



5. *a* can more than cover *b*:



6. Neither of *a* and *b* can fit inside the other:

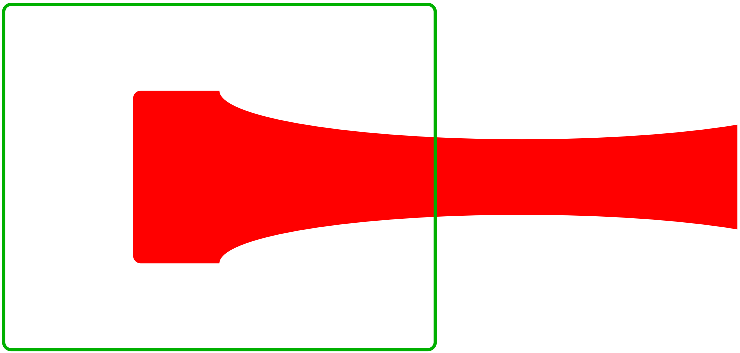


Continuity

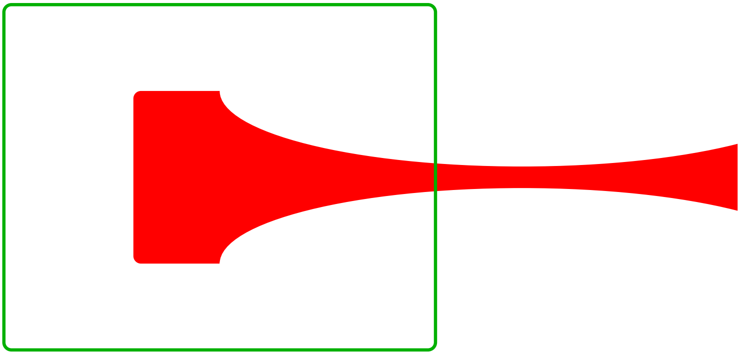
A Problem: What is meant by ‘Continuous?

The following slides seem to show a continuous direct transformation from PO to NTPP — which is not one of the links in the RCC8 Conceptual Neighbourhood Diagram.

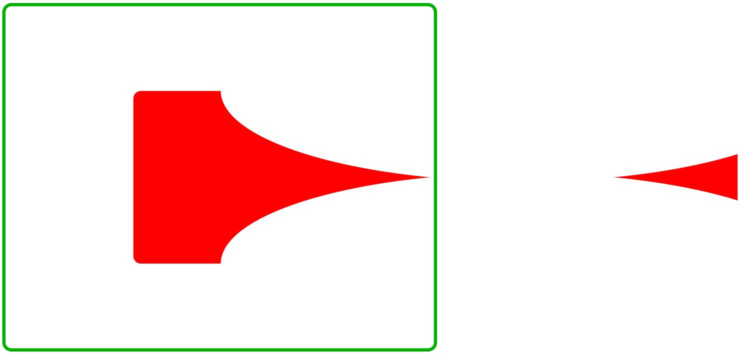
PO(red,green)



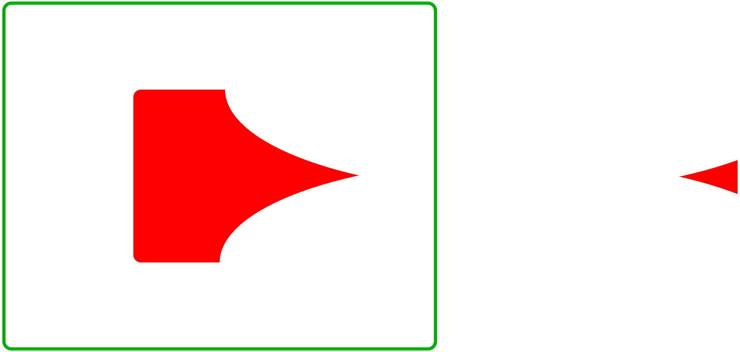
PO(red,green)



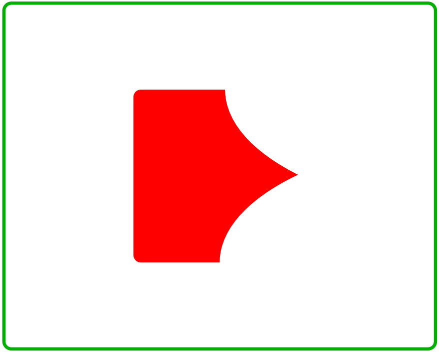
PO(red,green)



PO(red,green)



NTPP(red,green)



Continuous Change in Spatial Regions

Let Δ be a measure of the difference between two spatial regions.

We require Δ to be a **metric**, that is

1. $\Delta(R_1, R_2) \geq 0$
2. $\Delta(R_1, R_2) = 0 \leftrightarrow R_1 = R_2$
3. $\Delta(R_1, R_2) = \Delta(R_2, R_1)$
4. $\Delta(R_1, R_2) + \Delta(R_2, R_3) \geq \Delta(R_1, R_3)$

Suppose region $R = R(t)$ varies as a function of time.

Then *relative to the metric* Δ, the variation in R is **continuous at time t** so long as

$$\forall\epsilon > 0\,\exists\delta > 0\,\forall t'\,(|t - t'| < \delta \rightarrow \Delta(R(t), R(t')) < \epsilon).$$

Examples of metrics on (closed) regions

1. **Hausdorff distance**. The largest distance between any point in one region and the nearest point in the other:

$$\Delta_H(X,Y) \quad \stackrel{\Delta}{=} \quad \max\left(\sup_{x\in X}\inf_{y\in Y} d(x,y), \sup_{y\in Y}\inf_{x\in X} d(x,y)\right)$$

   where $d(x,y)$ is the distance between points x and y.

2. **Boundary-separation**. The Hausdorff distance between the *boundaries* of the two regions:

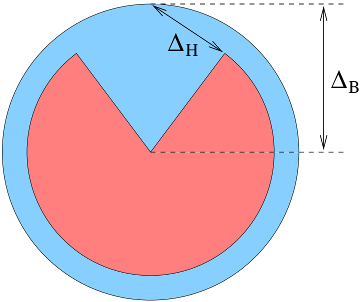
$$\Delta_B(X,Y) \quad \stackrel{\Delta}{=} \quad \Delta_H(\partial X, \partial Y).$$

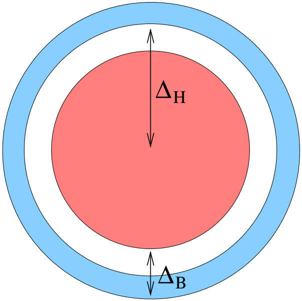
   where ∂X is the boundary of X.

3. **Size-separation**. The area (or volume in 3D) of the symmetric difference between the regions:

$$\Delta_A(X,Y) \quad \stackrel{\Delta}{=} \quad ||X \triangle Y||.$$

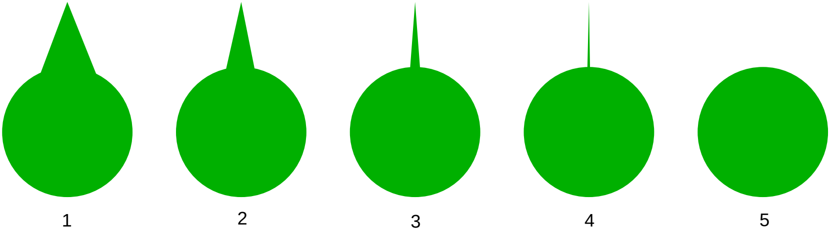
Comparison of two metrics





In the left-hand figure, the Boundary-separation is greater than the Hausdorff distance. In the right-hand figure it is the other way round. For *convex* regions the two measures always agree.

Continuity anomalies I



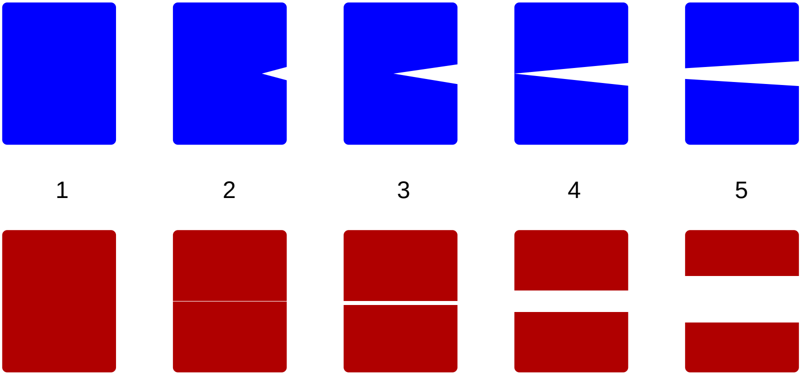
At $t = 5$, when the "spike" disappears, the change is continuous as measured by Size-separation, but discontinuous as measured by Hausdorff distance and Boundary-separation.

This is also the case for the transition from PO to NTPP illustrated earlier.

Continuity anomalies II

At $t = 5$, when the missing sector disappears, the change is continuous as measured by Size-separation and Hausdorff distance, but discontinuous as measured by Boundary-separation..

Continuity anomalies III



At $t = 4$, when the blue region splits in two, the change is Hausdorff-continuous, boundary-continuous, and size-continuous. At $t = 1$, when the red region splits in two, the change is Hausdorff-continuous and size-continuous, but not boundary-continuous.

The Theory of Dominance

Continuity and Conceptual Neighbourhood

The conceptual neighbourhood diagram for RCC8 relates discrete *qualitative* relations on spatial regions.

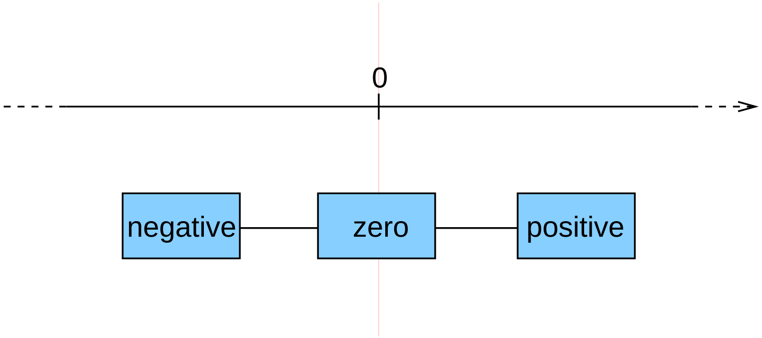
But these relationships are dependent on an underlying *continuous* reality.

The discrete space of qualitative relations that can be exhibited by a pair of regions can be derived systematically from a partition of an underlying continuous space of quantitative relations.

The basic idea

Consider the continuous space consisting of the real-number line $\mathbb{R}$

We divide it into three qualitative values 'negative', 'zero', 'positive', corresponding to the conditions $x < 0$, $x = 0$, and $x > 0$:



Conceptual Neighbourhood and Dominance

In the real-line example, ‘negative’ and ‘positive’ are both conceptual neighbours of zero, but not of each other.

These conceptual neighbourhood relations are asymmetrical in the following sense:

If a value changes continuously, it is possible for it to be positive during the interval (t_1, t_2) *and zero at* t_2*, but it is not possible for it to be zero during* (t_1, t_2) *and positive at* t_2*.*

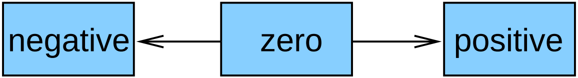
Suppose it is *positive* during (t_1, t_2) and *zero* during (t_2, t_3). Then the values *positive* and *zero* are “in competition” as to which of them holds at t_2. From the above continuity rule, the winner has to be *zero*.

Therefore we say that the value *zero* **dominates** the values *positive* and *negative*.

Dominance Spaces

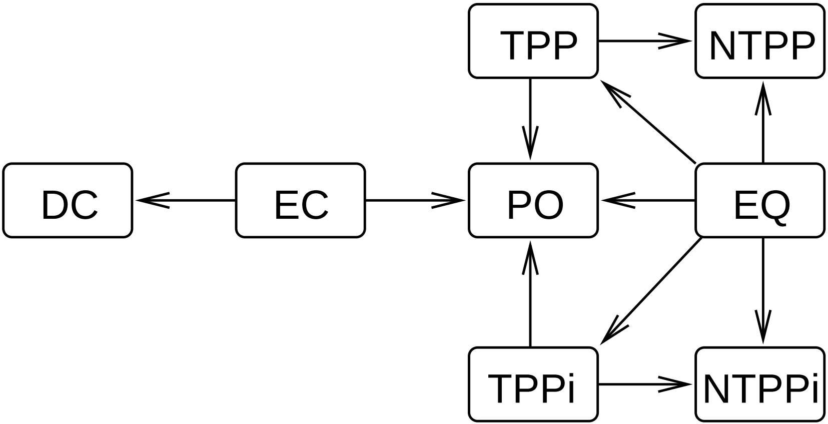
- A **dominance space** is a finite set of states Q together with an irreflexive, asymmetric relation $\succ$ (read 'dominates'), with the property that, for states $q, q' \in Q$, whenever q holds at one of the endpoints of an open interval over which q' holds, then $q \succ q'$.

Example. $(\{negative, zero, positive\}, \succ)$ is a dominance space, where $zero \succ negative$ and $zero \succ positive$. This can be illustrated diagrammatically as:



where the arrows indicate dominance.

RCC8 as a dominance space



The Product Theorem for Dominance Spaces

Let $(Q_i, \succ_i)$ $(i = 1, \ldots, n)$ be dominance spaces.

Then the product $(Q_1 \times \cdots \times Q_n, \succ)$ is a dominance space, where

$$(q_1, q_2, \ldots, q_n) \succ (q'_1, q'_2, \ldots, q'_n) \leftrightarrow$$
$$q_1 \succ_1 q'_1 \wedge q_2 \succ_2 q'_2 \wedge \cdots \wedge q_n \succ_n q'_n$$

Proof: See A. Galton, *Qualitative Spatial Change* (2000), pp.359–40.

Construction of RCC8

Let R_1 and R_2 be one-piece (self-connected) regions of co-dimension zero.

Let

$$p_1 = \frac{||R_1 \cap R_2||}{||R_2||} \qquad p_2 = \frac{||R_1 \cap R_2||}{||R_1||}$$

(so p_1 is the fraction of R_1 that falls inside R_2, and p_2 is the fraction of R_2 that falls inside R_1).

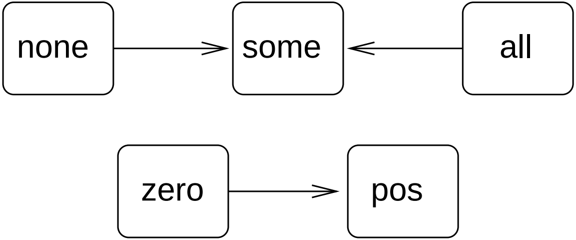
Let d be the minimum distance between a boundary point of R_1 and a boundary point of R_2.

Then the qualitative values of p_1, p_2, d uniquely determine the RCC8 relation between them.

Qualitative values of p_1, p_2, d

We consider the following qualitative values for the variables:
For p_1 and p_2: 'none' ($p_i = 0$), 'some' ($0 < p_i < 1$), 'all' ($p_i = 1$).
For d: 'zero' ($d = 0$), 'positive' ($d > 0$).

These qualitative values form little dominance spaces, as follows:

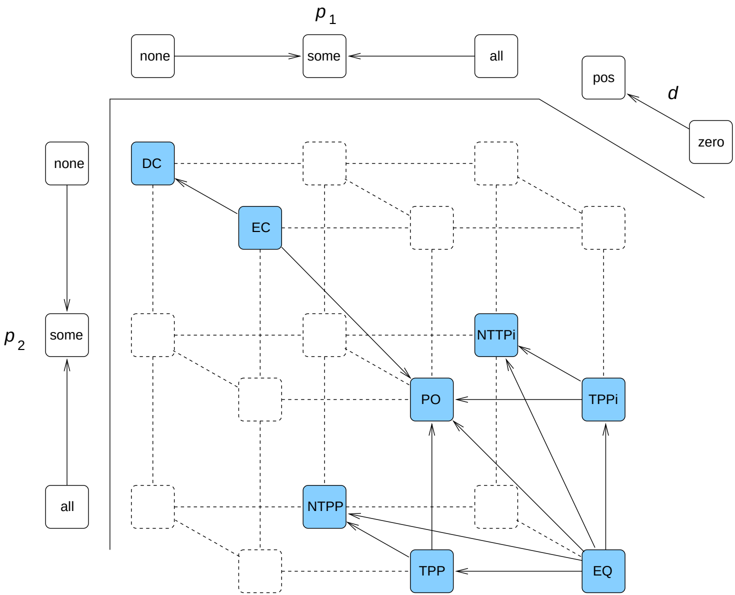


RCC8 relations determined by qualitative values of p_1, p_2, d

Then the RCC8 relation is determined as follows:

	p_1	p_2	d
DC	none	none	positive
EC	none	none	zero
PO	some	some	zero
TPP	all	some	zero
TPPi	some	all	zero
NTPP	all	some	positive
NTPPi	some	all	positive
EQ	all	all	zero

The product theorem now allows us to derive the dominance relations on RCC8 from the dominance relations for p_1, p_2, and d.


p 1
none
some
all
pos
d
zero
p 2
none
some
all
DC
EC
NTTPi
PO
TPPi
NTPP
TPP
EQ