

## **Workshop on *Relational Structures in Reasoning with Incomplete Information***

### ***Summary:***

The workshop presents, in some detail, the important role that relational structures play in some approaches to reasoning with incomplete information. It is divided into three parts, discussing various granular structures and related logics, in particular information systems and information logics.

### **Part I. (1 hr.)**

#### **On the Logical Aspects of Relational Granulation and Social Position Analysis**

*Churn-Jung Liao*, Institute of Information Science, Academia Sinica, Taiwan

Granular computing (GrC) is concerned with the processing of information granules, which are groups of objects drawn together by indiscernibility, similarity, proximity, or functionality. The process of forming information granules is called granulation. If the process is based totally on the attributes of the objects, it is called *functional granulation*, since attributes are mathematical functions from the set of objects to the set of values; if, in addition, the granulation process is also based on the relationship between objects, it is called *relational granulation*. Interestingly, social scientists have applied the techniques of relational granulation (albeit by different names) to positional analysis in social networks. Positional analysis of a social network tries to find *social role* or *social position* in terms of the similarities of the interaction patterns of the actors. Depending on the different patterns of relationships, different notions of positional equivalence have been proposed. These notions can be easily regarded as instances of relational granulation. In this talk, we consider three of the most important notions of positional equivalence – *structural equivalence*, *regular equivalence*, and *exact equivalence*. Recently, it was shown that social positions based on regular equivalence can be syntactically expressed as well-formed formulas (wff) in a kind of modal logic. Thus, actors occupying the same social position based on regular equivalence will satisfy the same set of modal formulas. By extending previous results, we can find logical characterizations of different positional equivalences and transform them into a functional granulation process. In this talk, we will start with an introduction to rough set theory--the basic theory of functional granulation. We then review several notions of positional equivalence in social network analysis and consider them as relational granulation from the GrC perspective. By using modal logics as a bridge, we then transform the relational granulation-based positional equivalences into functional granulation. Finally, we will conclude the talk with some practical considerations and open questions.

### **Part II. (2 hrs.)**

#### **Logics from Information Systems**

In this part, we address the representation and analysis of data structures known as Information Systems. Any such system consists of a collection of objects described in terms of their properties. A property is specified as a pair consisting of an attribute, and a value or a subset of values of this attribute. The value may even not be known. Such a

form of properties is a manifestation of incompleteness of information, and corresponds to different kinds of relational structures.

**Lecture i. (1+1/2 hrs.) Information Logics and their Relational Dual Tableaux**

*Joanna Golińska-Pilarek*, Institute of Philosophy, University of Warsaw

We will present several classes of logics originated in connection with information systems with incomplete information. First, we will recall the fundamental notions concerning information systems with incomplete information, information relations, and operators determined by these relations. We will discuss in detail two classes of relations referred to as indistinguishability relations and distinguishability relations. The indistinguishability relations reflect degrees of similarity and distinguishability relations correspond to degrees of dissimilarity. Next, we will present modal logics characterized by the classes of relational systems with some of these relations. We will study the models of the information logics based on the so called plain frames where each of the information relations is determined by the whole set of attributes of an information system, as well as the models based on relative frames consisting of the families of relations determined by all the finite subsets of the set of attributes of an information system. Finally, we will present relational dual tableaux for some typical information logics with modal operators determined by the information relations both from the group of indistinguishability relations and from the group of distinguishability relations. We show that relational dual tableaux are powerful tools for performing the four major reasoning tasks in information logics: verification of validity, verification of entailment, model checking, and verification of satisfaction in finite models.

**Lecture ii. (1/2 hr.) Dynamic Logics from Information Systems**

*Mohua Banerjee*, Department of Mathematics and Statistics, Indian Institute of Technology Kanpur

A representation of ‘flow of information’ in the context of information systems is discussed. A notion of ‘information update’ and resulting dynamic logics for single as well as multi-agent systems are presented. Comparisons with dynamic epistemic logics are made.

**Part III. (1 hr.)**

**More Relational Structures**

*Mohua Banerjee*, Department of Mathematics and Statistics, Indian Institute of Technology Kanpur

The third part of the workshop makes a survey of structures that emerge in connection with information granulation in general (related to GrC), and those in the special case of rough sets. Categories of granules on the one hand, and on the other, some (generalized) rough set structures such as Rauszer’s partition spaces, dynamic spaces, multiple-source and temporal approximation systems, property systems, uncertainty function based approximation spaces, and rough mereological structures, are presented. Different kinds of (multi-)modal propositional and predicate logics that result from these structures are given, and put in perspective.